Mathematics, Science and the Cambridge Tradition

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Abstract

In this paper the use of mathematics in economics, and the way in which mathematics contributes, or not, for economics to realise its potential as a science, will be discussed, by comparing two approaches to mathematics, a deductivist (algebraic) approach to mathematics, and a realist (geometrical) approach to mathematics. The differences between these approaches will be discussed in the context of the Cambridge tradition, while arguing that the work of key authors of this tradition was initially characterised by tension between their philosophical vision and their mathematical method, most notably in the case of Marshall, and that Keynes resolved this tension by focusing on uncertainty and probability relations instead of certainty and exact relations, while privileging a realist approach to the open and interconnected nature of economic phenomena.

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1. Introduction

Tony Lawson (2003a) argues that the use of mathematical deductivist methods became the defining feature of mainstream economics. These mathematical methods are deductivist in the sense that they presuppose constant conjunctions of the form "if event X, then event Y", where "event X" and "event Y" can refer to real or possible events. Lawson argues that these methods presuppose closed systems, which are systems in which constant conjunctions of the form "if event X, then event Y" are ubiquitous. However, since social reality is an open system, i.e., a system in which we do not observe scientifically interesting constant conjunctions of the form "if event X, then event Y", these mathematico-deductivist methods are argued to be inappropriate.

In this article I will discuss the mathematical methodology used in mainstream economics. A distinction will be made between a Cartesian algebraic (and *a priori*) approach to mathematics, where mathematics need not have any relation to reality, and a Newtonian geometrical (and *a posterior*) approach to mathematics, where mathematics is aimed at describing an underlying reality, and must be conjoined with a plurality of methods. I will then argue that mainstream economics is characterised by a Cartesian algebraic (and *a priori*) approach, which replaced the Newtonian (*a posteriori*) approach that was once dominant within the Cambridge tradition, in which there is a strong interplay between mathematics and reality.

This interplay, and the search for mathematical truths that can be known with certainty, led to a tension between a given philosophical conception of reality, and the search of certainty through mathematical methods, which characterises the work of Marshall (and exists also in Newton's work). This tension springs from the incompatibility between an ontological conception of reality where the latter constitutes an open system, and the limitations that mathematical methods which aim at certainty face in such situations.

While mainstream economists eliminated this tension by embracing a conception of mathematics where the latter is divorced from reality (following a Cartesian conception, which in mathematics culminated in Hilbert's program and the Bourbaki school, and in economics led to the general equilibrium analysis of Kenneth Arrow and Gérard Debreu), Keynes resolved this tension by focusing on the open nature of social and economic reality, while replacing the search for certainty in logical relations for a conception where statements can be known with a given degree of probability (given the available evidence), and where the probability distribution may not even be represented numerically (unless in case of exhaustive, exclusive and equal-probable alternatives). In Keynes' conception, the open nature of social and economic reality means that we typically are in a situation of uncertainty, and not of certainty, as will be argued here.

2. From marginalism to mainstream economics

While the expansion of mathematical methods in economics accelerated greatly in the twentieth century, the origins of mathematising projects can be found in the contributions of marginalists like Stanley Jevons and Leon Walras. Jevons (1888[1871]), for example, argued that since economics deals with quantities, it must use mathematics if it is to become a science, noting that the analysis of changes in quantities requires the use of differential calculus, which is extensively used today in mainstream economics in the study of optimising behaviour. And Leon Walras (1926[1874]) formulated a general equilibrium mathematical model of the whole economy, while also advocating the use of mathematical methods in economics.

The postulates of optimisation and equilibrium used by marginalist economists, in addition to other simplifying assumptions, have facilitated the extensive use of mathematical methods in economics in order to model the behaviour of optimising agents, and describe equilibrium states. Not all the authors who originated the marginalist revolution would have approved the use of mathematical methods as the main tool to model human behavior. Whilst Jevons and Walras were strong supporters of the use of mathematical methods, Carl Menger (also credited for being the founding father of Austrian economics) presented his work without mathematical methods, and the Austrian tradition he initiated provides a strong critique of the uncritical use of such methods – Friedrich Hayek (1948) is a prominent example.

Nevertheless, the development of mainstream economics has been characterised by an increasing use of mathematical methods when modeling human behaviour. The external environment faced by economics helped to promote the use of mathematical methods in economics. In fact, there was initially a strong opposition to the use of mathematical methods in social sciences, and this was a key reason why the work of Walras was not easily accepted initially. Jean-Baptiste Say, and the French Liberal school in general, together with influential mathematicians like Laplace, were much opposed to the mathematisation of social sciences, as Lawson (2003a) notes. The neglect of the mathematical work of Augustin Cournot by the French Liberal School is another example, in addition to the cold reception of Walras.

The rise of positivism during the first half of the twentieth century, together with the great success achieved by natural sciences where mathematical methods are extensively employed (such as physics), led however to the generalised belief that the use of mathematical methods is not only useful, but actually essential for economics to become a scientific discipline. In this sense, mathematical methods ceased to be only an instrument (to be used whenever it proved helpful for understanding economic phenomena), to become the very essence of the mainstream economic project – on this, see Lawson (2003a, chapter 10), who also argues that the mathematising tendency already existed within the Western culture, and benefited from a favorable environment for its development in the twentieth century.

One can thus question whether what we now term as mainstream economics is best characterised in terms of its mathematical methodology, and not in terms of substantive theories or postulates about the human agent (such as the postulate of optimisation). Mainstream economics is often identified with marginalism, the latter being characterised by the (Jevonsian) postulate of optimisation, and the (Walrasian) postulate of equilibrium. But one may question whether mainstream economics can still be identified with marginalism, or did it rather evolve to become something else, namely, a project characterised by the ubiquitous use of mathematical methods.

The distinction between mainstream economics as a commitment to mathematical methods, and marginalism as a commitment to postulates like optimisation (presupposing complete preferences) or the existence of an equilibrium state, may not look very important for practical purposes, for most mathematical methods employed in mainstream economics, when modeling economic behavior, are used in conjunction with marginalist postulates like optimisation, complete preference orderings, and the existence of an equilibrium state.

But this distinction is not irrelevant, for it is logically possible to use mathematical methods to model human behavior, and the economy, without presupposing optimisation or any of the marginalist postulates. Although there is certainly much overlap between marginalism and the current mainstream approach that evolved from the former, it is important to understand the transformations and shifts of emphasis of the mainstream project. The belief that mathematical methods are essential for economics to become a scientific discipline (as opposed to being just an useful instrument) must be carefully scrutinised, for this belief (and not mathematics *per se*) seems to have become an obstacle to pluralism in economics, and to the recognition of the usefulness of non-mathematical methods, as great as (if not greater than) marginalist requirements such as the prerequisite that human agency must consist in optimisation.

3. Mathematics and deductivism

There has been a wide debate within the fields of economic methodology and the philosophy of social sciences concerning the appropriateness of the mathematical methods used in economics, which are *deductivist* methods. Deductivism is a form of explanation that presupposes closed systems, where closed systems are systems characterised by constant conjunctions of the form "if event X, then event Y". Mathematico-deductivist methods presuppose closed systems, since they rely on a stable relationship between a set of dependent variables, and a set of independent variables, that is, they rely on quantitative functional relationships of the form y = f (x), where both "y" and "x" can be scalars, vectors or matrices.

However, closed systems do not exist spontaneously (except in celestial mechanics). Natural sciences, like physics, achieved great success through the use of functional relationships of the form y = f (x) because natural scientists construct laboratorial situations where a closed system is artificially generated, and constant conjunctions are observed. Since social phenomena cannot be insulated in the same sense as natural phenomena, the use of deductivist methods (that presuppose closed systems) has received much criticism within economic methodology and philosophy of science. At best, deductivist methods will be appropriate in particular instances where social phenomena approximates a closed system, but not in general.

As Lawson (2003a) argues, what is essential to science is the identification of the underlying structures, powers, mechanisms and tendencies which cause observed events. In natural sciences, closed systems, i.e., systems where event regularities of the sort ‘if event X then event Y’ are ubiquitous, are generated in well-controlled experimental conditions, so that mathematico-deductivist methods can be successfully applied, and underlying structures, powers, mechanisms and tendencies are insulated and identified through mathematico-deductivist methods.

In economics it is not possible to isolate human individuals like in the natural sciences. The social realm is thus irreducibly an open system, where event regularities are not ubiquitous, and the underlying structures, powers, mechanisms and tendencies cannot be insulated in the same way as in the natural sciences. Thus, Keynes stresses that mathematical methods like econometrics (correlation analysis) would only be successful under an atomistic conception of the economy, and Hayek (1948: 78) argues that a fundamental problem in economics, such as the use of knowledge in society, has been “obscured rather than illuminated (...) by many of the uses made of mathematics”, and by an “erroneous transfer to social phenomena of the habits of thought we have developed in dealing with the phenomena of nature”.

As Lawson (2003a) notes, the empirical regularities which are found in economics are not strict event regularities, but rather partial regularities, and our understanding of them will not benefit significantly from an elaborated mathematical treatment as if they were strict event regularities, such as the ones we find in laboratory experiments in the natural sciences, or in astronomy.

This methodological debate is extremely important for the assessment of the appropriateness of various types of quantitative methods in social research. If the social realm is an open system, where closed systems cannot be obtained as in the natural realm, the question arises as to whether economists, and social scientists in general, should concentrate more on non-deductivist quantitative methods, that do not postulate functional relationships of the form y = f (x), including the type of empirical analysis typically undertaken in descriptive mathematics and statistics. The distinction between a deductivist and a non-deductivist (descriptive) approach to mathematics is in fact a very important one, which will be further elaborated in the next section.

4. Cartesian and Newtonian approaches to mathematics

The distinction between a descriptive mathematics, which engages in an *a posteriori* analysis of of empirical reality, and a deductivist mathematics, which engages in *a priori* reasoning without regard to an underlying reality, ultimately bears upon the question of realism and mathematics. There have been various approaches to the role of realism in mathematics.

In some approaches, mathematics, as a product of pure reason, is to be developed with no reference to empirical reality. This is an approach much influenced by Cartesianism. For René Descartes, knowledge is achieved only through reasoning, and not through empirical study. Hence, there need be no connection between mathematical knowledge, and empirical reality.

In other approaches to mathematics, the use of mathematics when explaining reality is essentially a descriptive exercise where the development of mathematical methods depends upon the empirical reality under study. This was the case of the approach to mathematics of Isaac Newton, where mathematical methods were meant to describe empirical observations in a realist way, and were in fact applied to explain a reality which was a closed system, namely celestial mechanics.

As Michael Atiyah (2005: 654-657) explains, the differences between Newton and Descartes go back to a more profound distinction within mathematics, between geometry and algebra. Geometry is concerned with space, and with the study of physical objects, while algebra is concerned with operations, that is, the transformation of elements of a set (such as numbers) into other elements through operations. In this sense, geometry is concerned with space, while algebra, by studying transformation through operations, is concerned with time.

Geometry goes back to the Greeks (and even beyond, to the Babylonians, who influenced Greek mathematicians), while algebra goes back to the Arabs and Indians – see Atiyah (2005: 655). Newton was essentially a geometer, who thought of geometry as grounded on empirical mechanics, and the study of physical motion. Descartes was an algebraist, who introduced algebraic coordinates into geometry, attempting thus to reduce geometry to algebra, in what is termed "analytical geometry".

Descartes introduced the notion of a "variable", and in this way brought time to mathematics, addressing it in an algebraic way. Newton, on the other hand, attempted to approach time in a geometrical way, in his differential calculus. The differences between Newton and Leibniz regarding differential calculus are also related to the distinction between geometry and algebra. While Newton's differential calculus was an attempt to integrate time into a geometrical approach to mathematics, Leibniz, who like Descartes was an algebraist, had the aim of "formalising the whole of mathematics, turning it into a big algebraic machine" (Atiyah 2005: 655).

In fact, there is a tendency for algebraists to have an algorithmic approach to mathematics, while geometers tend to emphasise not only the role of physical reality, but also of intuition. Atiyah (2005: 655) points out Henri Poincaré and David Hilbert as contemporary successors of Newton and Leibniz, respectively, with the group of French mathematicians writing under the name "Bourbaki" being also an influential school following the algebraic approach.

While in the "geometrical" approach mathematics is concerned with empirical reality (such as space) and relies upon our human intuition and creativity when perceiving an external reality (a point much emphasised by Poincaré), in an exclusively "algebraic" approach mathematics becomes an algorithmic, formalistic, rationalistic and deductivist exercise, divorced from reality, and concerned only with possible realities that can be addressed in a formalistic and deductivist way, as it was the case with the "Bourbaki" school. It is the later approach to mathematics that characterises mainstream economics, which uses mathematics in a deductivist way, in the sense that it attempts to reduce economics to a formalistic approach that presupposes closed system regularities.

These two approaches reflect two different conceptions of change. The Cartesian algebraic approach explains change in algorithmic functional terms, through algebraic operations, and a formalisation which is rationalistic, in the sense that it is obtained through "pure" reasoning, as Descartes did. In the Cartesian algebraic approach, mathematics is detached from reality and from empirical notions like space, which were fundamental to Newton's mathematical approach to the natural sciences. Hence Descartes' dualism between *res cogitans*, where the key to knowledge is, and *res extensa*, i.e., extension in space, which is proved to exist only through reason.

Geometrical explanations of change, on the other hand, are grounded on notions like "distance", "relation" and "structure", which are "geometrical" notions, in the sense that they are grounded on our conception of space. In fact, even time can be understood in geometrical terms, as a relation (or a distance) on another dimension.

Central contributions to natural science, like Newton's, had a geometrical approach to mathematics, where creativity, intuition and reality were more important than formalism or deductivism. Algebra, no doubt a powerful tool, is sterile when addressed only in a rationalistic and deductivist way. In this sense, the uncritical use of mathematico-deductivist methods which characterises mainstream economics not only springs from a misunderstanding on the conditions under which the use of those methods is appropriate, as its attempt to imitate natural sciences also fails to meet the spirit of the contributions of natural scientists like Newton.

As Lawson (2003a: 263-264) notes, the sucesses of Newton's mathematics were interpreted in France in Cartesian terms, as a proof of the usefulness of mathematics in general, and not as evidence that mathematics can be most useful when it takes into account the nature of the reality under study. The Cartesian approach to mathematics that prevailed in France shifted the attention towards the mathematical formal structure of Newton's theory, and neglected the way in which reality influenced the development of such theory.

The French Bourbaki school, which Atiyah (2005: 655) identifies as the twentieth century heir of the Cartesian algebraic approach, is an example of this type of approach, which inspired central contributions to mainstream economics like Gérard Debreu (1959), in a context where economics (like mathematics, following the "algebraic", or "*a priori*", approach), became concerned with possible realities, and not with reality itself – see Dow (2003) or Lawson (2003a: 271-273).

This was the Cartesian approach to mathematics towards which Say was very agressive with, and it was the approach which ultimately accepted the work of Walras and the marginalists, not by attempting to combine it with classical political economy as Marshall did, but in fact rejecting classical political economy, leading to the uncritical use of mathematico-deductivist methods that characterises mainstream economics. It is thus no surprise that contributions characterised by a mathematico-deductivist approach, of authors like Debreu (1959), are considered as the more elaborate and admirable achievement of economic theory within mainstream economics.

5. Mathematics and the Cambridge Tradition

While mainstream economics is best characterised in terms of a mathematico-deductivist methodology, it is true that most mainstream economics subscribe also to the theoretical principles of marginalism, which pioneered the use of mathematical methods in economics. Marginalism is often identified with what is called *neoclassical economics*. This identification, together with the fact that there is much overlap between mainstream and marginalism, leads many to refer to "neoclassical economics" as the dominant economic perspective.

However, this use of the term "neoclassical economics" is a misleading one. After the marginalist revolution, Alfred Marshall (1890) attempted to make marginalist theory compatible with classical political economy, leading to an approach which was thus termed by Thorstein Veblen (1900) as neo*classical*, in order to distinguish it from the approach of Menger (which inspired Austrian economics). In fact, Keynes (1936) considered Marshall's neoclassicism to be a continuation of classical economics. In so doing, Marshall initiated the Cambridge economic tradition, shaped by his contribution (see Harcourt, 2003), while Marshall's (1890) *Principles of Economics* became the canonical economics textbook not only in Cambridge, but in many other economic departments.

The fact that Marshall attempted to establish continuity with Smith, Ricardo and Stuart Mill does not mean that he succeeded. In fact, as Pierangelo Garegnani (1998, 2005) explains, Smith and Ricardo did not analyse value in terms of the interaction between supply and demand, as Stuart Mill and Marshall did. It was Piero Sraffa who, after having criticised Marshall's approach – Sraffa (1925, 1926) – recovered the classical theory of value of Smith, Ricardo and Marx, beginning to develop it after the end of 1927, in a project which led to Sraffa's (1960) revival of classical political economy.

Most of Sraffa's work remained unpublished. In fact, as Annalisa Rosselli (2005: 405) notes, Sraffa writes, in a letter to Charles Parrish Blitch (dated October 6, 1975), that "in economic theory the conclusions are sometimes less interesting than the route by which they are reached", which signals how Sraffa was more concerned with the process of discovery rather than with the publication of results. But if we take Smith and Ricardo as the key authors of classical political economy, rather than Stuart Mill's version of it, then it is Sraffa, and not Marshall, who really engages in a continuation of classical political economy.

The Cambridge economic tradition is divided into various streams, including not only the Marshallian stream, but also the Keynesian and Sraffian streams, which emerged with the critique of Marshall undertaken by Keynes and Sraffa. But there is one aspect of Marshall'w work which remained throughout the Cambridge economic tradition, namely Marshall's realist approach, which contrasts with Walras's use of mathematics. As Joseph Schumpeter argues:

"Just as Walras, more than any other of the leaders, was bent on scraping off everything he did not consider essential to his theoretical schema, so Marshall, following the English tradition, was bent on salvaging every bit of real life he could possibly leave in." (Schumpeter 1994: 974)

Thus, while Marshall thought that mathematical analysis should be left out of the main text of a book (as he did in his work), mainstream economics evolved in a way where mathematical analysis became the central aspect of any mainstream economics textbook.

In fact, the use of mathematics in Marshall, and in the Cambridge tradition, is very different from the mainstream use of mathematics. Marshall and the Cambridge tradition were much influenced by the Newtonian approach to mathematics, which was very different from the Cartesian approach. It is true that after 1815 at least, the Cartesian approach was gaining ground in Cambridge too. But as Simon Cook (2009) explains, the Cambridge Mathematical Tripos that Marshall undertook still contained many elements which remained from the Newtonian approach, such as an emphasis on geometrical and mechanical problems, rather than on symbolic algebra.

Newton was concerned with providing a foundation for science which takes into account the nature of reality. However, Newton was also very concerned with achieving certainty. As Niccolò Guicciardini (2006: 1736) explains, "from the early 1670s [Newton] expressed his distaste for the probabilism and hypotheticism that was characteristic of natural philosophy", since mathematics should provide certainty to natural philosophy.

However, as Guicciardini (2006: 1736) also notes, there was a tension in Newton's perspective, for while "Descartes was the champion of an impious mechanistic philosophy (...), Newton conceived himself as a restorer of an ancient, forgotten philosophy according to which nature is always open to the providential intervention of God", and thus, this "led Newton into a condition of strain, since his philosophical values were at odds with his mathematical practice, which was innovative, symbolical, and – pace Newton – deeply Cartesian."

Guicciardini (2006: 1736-1737) continues:

"Several hitherto unexplained aspects of Newton's mathematical work are related to this condition of stress and strain that characterizes his thoughts on mathematical method. Why did Newton fail to print his method of series and fluxions before the inception of the priority dispute with Leibniz? Why did he hide his competence in quadratures when writing the *Principia*, which are written mostly in geometrical style? Even though there is no single answer to these vexed questions, I believe that Newton's conviction that the analytical symbolical method is only a heuristic tool, not 'worthy of public utterance', can in part explain a policy of publication which was to have momentous consequences in the polemic with Leibniz."

There is an interesting similarity between the explanation that Guicciardini (2006, 2009) gives for Newton's postponement of the publication of his work, and the explanation that Stephen Pratten (1998) provides for Marshall's failure to finish the second volume of his *Principles*. Pratten (1998) argues that it was the tension between Marshall's vision of reality, and his use of a mathematical method which was inappropriate for analysing such reality, that prevented Marshall from achieving a satisfactory second volume, that would reconcile the statical method used in the first volume with the dynamical approach that was to be developed in the second volume.

In both cases, of Newton and Marshall, there is a tension between an underlying philosophical vision, and method. This tension led Newton to continuously revise his methods, which were initially much inspired in Descartes. A similar tension led Marshall to argue that the Mecca of all economists lies in evolutionary biology, and not in physics, which inspired the conception of science that underpins the first volume of Marshall's *Principles*.

John Maynard Keynes (1936) later criticised the Marshallian framework of the *Principles*, as developed by Arthur Pigou (1920), initiating the Cambridge Keynesian tradition (on which see Harcourt, 2003, or Pasinetti, 2005). In fact, after Keynes' (1936) contribution, the Cambridge economic tradition became divided between what may be termed as the Cambridge 'welfare' tradition, which followed Marshall and Pigou, and the Cambridge Keynesian tradition, which rejected the neoclassical framework of Marshall and Pigou. But Keynes' overall conception contrasts not only with the Marshallian neoclassical framework, but also with Newton's search for certainty.

In fact, Keynes is the first influential author of the Cambridge economic tradition to successfully transcend the tension generated by the search for certainty, by resorting to an original conception of probability. Keynes (1921) argues that even when we cannot establish an exact law that relates two propositions with certainty, there is still a logical relation between the two propositions, namely the probability relation, wherein a given conclusion is alyways related to a given premise with a given probability.

The numerical value of this probability can only be known in the case of exclusive, exhaustive and equi-probable alternatives, as Tony Lawson (1985b) and Anna Carabelli (1985) explain. This led Keynes (1921) to replace certainty for uncertainty, and exactness for probability, leading to a radically new conception – see Lawson (1985a). Carabelli (1985: 167) writes that "Keynes seemed to see his work on probability as a sort of anti-*Discours de la méthode*, based on probability, ordinary discourse and common sense rather than on certainty and on analytical reason". Thus, Keynes's conception was a definitive rejection of the Cartesian project.

It is important to understand the differences between Keynes' conception of probability, and the mathematical approach to probability of authors like Bernoulli, Pascal and Huyghens. As Carabelli (1985: 159) explains, Keynes was very unhappy with the conception of probability of authors like Bernoulli, Pascal and Huyghens, who assumed the principle of equal probabilities when there was no sufficient reason to think otherwise.

This assumption was fundamental to enable the mathematical treatment of probability, but was in fact assuming a numerical value without any basis for such, an assumption which Keynes found unsatisfactory. Newton was also against this mathematical approach to probability which characterises the work of Bernoulli, Pascal and Huyghens, due to the lack of a basis for such a mathematical treatment. As Guicciardini (2006: 1736) explains, this led Newton to criticise the mathematical theory of probability:

"by the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a science of nature supported by the highest evidence." (Newton translated and cited in Shapiro 1993, 25)

So while Newton was very hostile against the mathematical treatment of probability of his time, of authors like Bernoulli, Pascal and Huyghens, Keynes treatment of probabilities, and rejection of such mathematical treatment of probability, resolves the tension created by the search for mathematical certainty, avoiding a "condition of strain" that characterised the Cambridge mathematical tradition inspired by Newton, and subsequently the Cambridge economic tradition inspired by Marshall (which, as noted above, had a mathematical approach much influenced by Newtonian mathematics too). Keynes achieves this through a conception were probability is grounded on ordinary language and common sense, and not on Cartesian certainty.

Keynes' idea of grounding probability on ordinary language was in fact similar to Newton's approach to mathematics in many ways. As Guicciardini (2006) notes, Newton was satisfied only when his conclusions could be supported by geometrical demonstrations which were "worthy of public utterance". For example, concerning the squaring of the curves (what today we call integration, following Leibniz), Newton writes:

"After the area of some curve has thus been found, careful considerations should be given to fabricating a demonstration of the construction which as far as permissible has no algebraic calculation, so that the theorem embellished with it may turn out worthy of public utterance." (Newton, as cited in Guicciardini 2006: 1734-1735).

The use of ordinary language that is "worthy of public utterance" continued to play a great role in the Cambridge tradition in the twentieth century, in authors like Moore, Keynes and Wittgenstein, who emphasised the importance of common sense – see Coates (1996) for a discussion. Keynes was concerned with grounding his work on probability on ordinary language, as Newton also was concerned with geometrical demonstrations "worthy of public utterance".

This approach ultimately led Keynes to a realist approach to economics, in which Keynes criticised the use of mathematical methods in economics, arguing that economic phenomena is not homogeneous through time, and thus mathematical methods which presuppose the contrary will be inappropriate – see Lawson (2003a). If we take a Newtonian approach to mathematics to mean a concern with the nature of reality, Keynes can be said to maintain what was termed above as a Newtonian approach to mathematics, in the sense that, for Keynes, mathematics must conform to the analysed reality, in opposition to a Cartesian approach to mathematics where reality (and empirical data) are neglected, which characterises mainstream economics.

Based on Feynman's (1965) notion of "Babylonian mathematics", and on Keynes' (1963) writings on Newton, Sheila Dow (1990) uses the term "Babylonian method" to designate the methodology of the Keynesian tradition, arguing that this method, charaterised by the use of multiple strands of argument with different starting points, was also used by classical political economists from Smith to Marx. Dow (1990) argues that mainstream economics, on the other hand, is characterised by Cartesian deductivist methods, which are only a particular case of the Babylonian methodology.

For Dow, like for Lawson, mainstream economics is grounded on the albegraic deductivist approach to mathematics, while classical political economy and the Cambridge tradition are characterised by methodological pluralism where, I would add, even mathematics is used in a geometrical (Newtonian) approach. This is in fact the key difference in the approach to mathematics that underpins mainstream economics, and the one that underpins other approaches from classical political economy to the Cambridge economic tradition.

6. The mathematisation of economics

In the middle of the twentieth century, John Maynard Keynes and Jan Tinbergen engaged in a debate concerning the merits of econometrics, which was then an emerging field (a debate in which Keynes criticized the use of econometrics, and the mathematization of the discipline) – see Tony Lawson (1985b). It is also in this time that game theory emerges, first with the contribution of John von Neumann and Oskar

Morgenstern (1944), and soon after with the contribution of John Nash (1951). This is also the moment where Milton Friedman (1953) argues that realism is not necessary for economic theory. And it is at this time that Kenneth Arrow and Gérard Debreu (1954) develop Walrasian general equilibrium analysis.

The approach to mathematics that became dominant in this process was the Cartesian algebraic approach, where mathematics becomes divorced from reality, and concerned only with possible realities that can be addressed in a formalistic and deductivist way, as it was the case with the "Bourbaki" school that became a very influential school of mathematics in the twentieth century. As Tony Lawson (2003) explains, it is the later approach to mathematics that characterizes mainstream economics, which uses mathematics in a deductivist way, in the sense that it attempts to reduce economics to a formalistic approach that presupposes closed system regularities. If the influence of mathematicians like Von Neumann or Nash was pointing in this direction, the contribution of Gérard Debreu, and his formalization of general equilibrium theory, is one of the clearest examples of this approach. Thus Lawson writes:

"Although Debreu's *Theory of Value* was produced after his move to the US Cowles Commission in the 1950s, Debreu was very much a product of the French Bourbaki 'school' (a group of French mathematicians who argued that mathematical systems should be studied as pure structures devoid of any possible interpretations). It was at the Ecole Normale Supérieure in the 1940s that Debreu came into contact with the Bourbaki teaching. And once trained in this maths, but with his interests aroused by economics, Debreu sought a suitable location to pursue an interest in reformulating economics in terms of this mathematics. It is perhaps not insignificant that his move to the Cowles Commission coincided with the latter's effective acceptance of Bourbakism." (Lawson 2003: 273)

And even when attempts are made to apply mathematics to reality within mainstream economics, as in econometrics, we find a wide gap between econometric theory and econometric practice, as Edward Leamer (1983: 37) points out, where "hardly anyone takes anyone else's data analysis seriously" – see Lawson (2003: 11) for a discussion

The consequences that Marx extracted from classical political economy concerning the distribution of the surplus also contributed not only for the definite abandonment of classical political economy, as a reaction to Marx, but also for an acceptance of the Bourbaki approach to mathematics:

"In particular the emergence of McCarthyite witch-hunts in the context of the Cold War significantly affected the developments in which we are interested. In this climate, the nature of the output of economics faculties became a particularly sensitive matter. And in such a context, the project of mathematising economics proved to be especially attractive. For it carried scientific pretensions but (especially when carried out in the spirit of the Bourbaki approach)" (Lawson 2003: 274)

This approach to mathematics divorced from reality contrasts with the geometrical approach of Newton, which was very influential in Cambridge in a time when the Cartesian algebraic approach (concerned with abstract symbols, rather than concrete reality) was increasingly dominant in the European continent. In Marshall's time, the Cartesian approach had finally been gaining ground in Cambridge too, but the Mathematical Tripos Marshall undertook still contained mostly elements of the

Newtonian approach, such as an emphasis on geometrical and concrete mechanical problems, rather than on symbolic algebra – see Simon Cook (2009).

Thus, if Marshall and Keynes were against an uncritical use of mathematical models in economics (as we can see by Marshall's belief that mathematics should be left to appendixes, and Keynes exchange with Tinbergen), they would certainly have rejected the Cartesian use of mathematics in mainstream economics, and its uncritical use of mathematical models that were pioneered by marginalists like Jevons and Walras (and used before by economists like Cournot), and which became central to mainstream economics in the twentieth century. Even Sraffa (1960) insisted on presenting his work with the minimal use of symbolic algebra, or abstract mathematical symbols, contrarily to the recommendations he received.

Given the increasing importance attributed to mathematical-deductivist methods, Lawson (2006) argues that mainstream economics is indeed now best characterized by the uncritical acceptance of mathematical-deductivist methods. It is important to note that Lawson is not criticizing the use of mathematics in general, but only a particular type of use, namely the Cartesian use of mathematics, disconnected from a concern with reality. Thus, Lawson writes:

"It is not, and has never been, my intention to oppose the use of formalistic methods in themselves. My primary opposition, rather, is to the manner in which they are everywhere imposed, to the insistence on their being almost universally wielded, irrespective of, and prior to, considerations of explanatory relevance, and in the face of repeated failures. (Lawson 2003: xix)"

Thus, mainstream economics became characterized by a deductivist approach to mathematics divorced from reality, which constitutes now its main characteristic in the academic world.

7. The philosophy of the Cambridge economic tradition

A question which remains concerns the philosophical vision of the Cambridge economic tradition, which generated the tension with mathematical methods that was transcended by Keynes (1921). In fact, a strong interaction between philosophy and economics has continually occurred in the Cambridge tradition, not only through the influence of philosophers like Sidgwick on Marshall, but also throughout the twentieth century in the interchange between philosophers like G. E. Moore, Bertrand Russell, A.N. Whitehead, Frank Ramsey and Ludwig Wittgenstein on the one hand, and economists like John Maynard Keynes and Piero Sraffa on the other hand.

The influence of Moore's and Whitehead's organicism on Keynes (on which see Dow 1990, 2003), the exchanges on probabability between Keynes and Ramsey (on which see Runde 1994), or the role Sraffa played in Wittgenstein's change of perspective (on which see John Davis, 1988, 2002, or Amartya Sen, 2003) are examples of this interaction – see also Coates (1996).

The tension between reality, and a method which cannot completely describe it, is a more specific instance of a more general tension, namely that between the finite and the infinite. As Dow (1990) explains, this is well captured by Whitehead, who argues that this tension generates a continuous process, making reality intrinsically dynamic:

"By means of process the universe escapes from the limitations of the finite. Process is the immanence of the infinite in the finite; whereby all bounds are burst, and all inconsistencies dissolved" (Whitehead, 1938: 75)

As Dow (1990: 148) explains when commenting on the passage above, the Cartesian approach (which characterises mainstream economics) leads to finite and dichotomous categories which are not necessarily consistent with the reality under study which, as Whitehead (1929) explains, is an organic process.

Whitehead (1929) attempted to reconcile the diversity existing in the permanent flux of empirical events with the possibility of obtaining certain knowledge, by arguing, following Plato, that empirical events are the actualisation of eternal (Platonic) forms, which are a geometrical notion (this being the reason why one would have to be a geometer to enter Plato's Academy). According to Whitehead, in an approach which is reminiscent of the Cambridge Platonists, our knowledge grasps these forms.

The atmosphere in which Keynes started his work was in fact much influenced by Plato, who was a central author not only to Whitehead, but also to Moore. Some authors, like Carabelli (1988) attribute to Keynes an Aristotelian perspective instead, the reason being that Keynes was concerned with change, while Platonic forms are eternal and unchanging forms. Nevertheless, Whitehead's interpretation of Plato also allows for the conceptualisation of change and process. In fact, it was precisely in order to explain change that Whitehead (1938) resorts to the Platonic realm of ideas, and to the immanence of the infinite in the finite.

To some extent, Whitehead remained too in a "condition of strain" between his mathematical approach and philosophical vision, which led him to remain somewhat dualist, as Lawson (2003b) explains. For example, he attempted, together with Russell, to formalise mathematics. Although Whitehead adopted a conception of reality as an organic process, his philosophy remained still somewhat dualistic, since he did not acknowledge any organic relation between eternal forms and events, as Lawson (2003b: 114) explains.

Although Moore and Whitehead, who influenced deeply Keynes' organicist conception, were much influenced by Plato, our focus here is essentially on the philosophical concepts they brought to the Cambridge tradition, for example organicism, which Whitehead developed into a notion of organic process.

Within this context, how can we then characterise the underlying philosophy of the Cambridge tradition? I believe that the notions of *open system* and *internal relation*, whch are being developed by Tony Lawson and the Cambridge Social Ontology Group, can play a key role in the interpretation of the method of these various authors of the Cambridge economic tradition.

As Lawson (2003a) explains, aspects or itens are internally related when they are what they are, or can do what they do, in virtue of the relation in which they stand to each other, that is, the internal relations are constitutive of the related entities. The notion of *internal relation* can be argued to be the fundamental notion of the Cambridge tradition, which helps us to understand the notions of organic whole, and process.

Harcourt (2003) systematises the three key characteristics of the Cambridge economic tradition as being the following: the whole is more than the sum of the parts; agents act in a context of inescapable uncertainty; and the existence of a plurality of languages. The second and third element that Harcourt identifies can be seen as a consequence of the first. The fact the reality is deeply interconnected, and that we can only grasp a part of this reality, leads naturally to the existence of inescapable uncertainty for each agent who can perceive only a part of the whole.

Each part is an *open system* related to the whole, and if the whole is not finite, the whole itself will be also an open system, where uncertainty is always present. This means also that one single method or language is not sufficient to address such a complex reality. Rather, various languages and methods are needed to capture each part of such a complex organic whole, which is the third point noted by Harcourt.

Organic wholes occur when given interrelated entities are mutually constituted by their relations; and a process can be best understood as a relation through time. While the mathematic-deductivist methods of mainstream economics presuppose an atomistic ontology, the Cambridge tradition presupposes a relational ontology, where relation refers to spatial relations, and temporal relations.

The organicism, or relational ontology, of Moore and Whitehead, was very important to Keynes, who developed an approach to economics where the whole is more than the sum of the parts, as Harcourt (2003) explains. This leads Keynes to a macroeconomic perspective, which cannot be reduced to microeconomics precisely because of the organic connections between phenomena. An organic conception cannot be captured by a method which presupposes atomism, as Keynes argued when criticising the use of mathematical methods that presuppose isolated (legal) atoms, and presuppose that economic phenomena are homogeneous through time – see Lawson (2003a).

The notion of organic processes was not developed entirely by Moore and Whitehead, however. The general idea was already part of the Cambridge tradition, as can be seen in Marshall's suggestion that evolutionary biology contains the key to understand change. In fact, the notion of organic processes is inspired in evolutionary biology, and not on the physics and mechanics which inspired Marshall's first volume of the *Principles*, and much of mainstream economics.

Effectively, Marshall's own philosophy was much influenced by Hegelianism, and the idea of internal relation, as Cook (2009) explains – see also Marshall's early writings, edited by Whitaker (1975). The notion of internal relation was subject to much critique in Cambridge, because it was initially interpreted in an idealist Hegelian fashion. Thus, central authors like Bertrand Russell were led to an atomist perspective instead. However, the ontological underpinnings of the notion of organic process remained in the contributions of many authors, and are only now being elaborated more systematically, for examply by Lawson, through a realist (rather than idealist) ontological perspective.

8. Concluding remarks

Mainstream economics has its origins in the mathematical approach of Walras and Jevons, and in the marginalist revolution which, despite the efforts of Menger and Marshall, ultimately led to the abandonment of a realist approach to economics. The crucial moment occurred in the twentieth century however, with the econometric revolution, the appearance of game theory, and general equilibrium models. The latter, and especially the work of Debreu (1959), contains the more elaborated version of the Walrasian project, which maintains a Cartesian (algebraic) approach to mathematics, and contrasts with the Newtonian (geometrical) approach to mathematics.

The great success of natural sciences like physics led mainstream economists to believe that mathematico-deductivist methods are essential for economics to become a science. But while natural scientists construct closed systems in laboratory experiences so that physical and chemical phenomena may be exactly measured through mathematico-deductivist methods, mainstream economists posit that human agents behave in a rational, exact and predictable way, so that similar mathematico-deductivist methods can similarly be used in a supposedly ‘scientific’ way, while ignoring the nature of the underlying reality.

Central authors of the Cambridge economic tradition, like Marshall and Keynes, adopted a different approach to mathematics, which is not deductivist, and existed already in Newton. But Marshall’s method, based on physics, still constrained him from developing consistently his insights regarding evolutionary biology. The implicit ontology of evolutionary biology, which is an ontology of organic processes, was developed only later, and is underpinned by the ontological categories of *structure* (implicit in Moore’s organicism) and *process* (elaborated by Whitehead). The notion of *internal relation* may be seen as an even more fundamental one, for structures can be seen as relations in space, and processes may be seen as relations through time. These categories, of structure, process and relation have been more recently developed by Lawson (2003a), and are fundamental for our understanding of the philosophical underpinnings of the Cambridge economic tradition.

Like in every tradition, not all the authors identified with the Cambridge tradition adopted this ontology. Bertrand Russell, an admirer of Democritus and the atomists, is a prominent example of an exception (or, more correctly, of someone who abandoned the tradition). The current orientation of Cambridge economics teaching is also heading away from this tradition, as Harcourt (2003) notes. But if a coherent framework is to be found in the Cambridge tradition, and in the Cambridge economic tradition in particular, it can be found especially at an ontological level, within a conception where reality is an internally related totality in motion, i.e., a *structured process*, where other characteristics, like a pluralistic (Babylonian) methodology, are a consequence of an attempt to analyse a diverse reality, which leads to the need of diverse methods. This approach contrasts with mainstream economics, where unity is found at the level of method, as Lawson (2003a) argues.

References

Arrow, K. J. and Debreu 1954, G. “Existence of an Equilibrium for a Competitive Economy,” *Econometrica*, 22, 265-90.

Atiyah, M. 2005, *Collected Works, Volume 6*, Oxford, Oxford University Press.

Carabelli, A. 1985, “Keynes on Cause, Chance and Possibility”, *in* T. Lawson and H. Pesaran (eds.), *Keynes’s Economics: methodological issues*, London, Croom Helm, 151-180.

Carabelli, A. 1988, *On Keynes’ Method*, London, Macmillan.

Coates, J. 1996, *The Claims of Common Sense: Moore, Wittgenstein, Keynes and the Social Sciences*, Cambridge, Cambridge University Press.

Cook, S. 2009, *The Intellectual Foundations of Alfred Marshall's Economic Science: A Rounded Globe of Knowledge*, Cambridge University Press: Cambridge.

Davis, J. 1988, “Sraffa, Wittgenstein and neoclassical economics”, *Cambridge Journal of Economics*, 12, 29-36.

Davis, J. 2002, “Gramsci, Sraffa, Wittgenstein: philosophical linkages”, *European Journal of the History of Economic Thought*, 9, 384-401.

Debreu, G. 1959, *Theory of Value: an axiomatic treatment of economic equilibrium*, New York, Wiley.

Dow, S. 1990, "Beyond Dualism", *Cambridge Journal of Economics*, 14, 143-157.

Dow, S. 2003, "Understanding the relationship between mathematics and economics", *Journal of Post Keynesian Economics*, 25, 547-560.

Feynman, R. P. 1965, *The Character of Physical Law*, Cambridge MA, MIT Press.

Friedman, Milton (1953), *Essays in Positive Economics*, Chicago: Chicago University Press.

Garegnani, P. 1998, "Sraffa: The Theoretical World of the 'Old Classical Economists'", *European Journal of the History of Economic Thought*, 5, 415-429.

Garegnani, P. 2005, "On a turning point in Sraffa's theoretical and interpretative position in the late 1920s", *European Journal of the History of Economic Thought*, 12, 453-492.

Guicciardini, N. 2006, "Method versus calculus in Newton's criticisms of Descartes and Leibniz", *Proceedings of the International Congress of Mathematicians*, *Madrid, Spain*, 1719-1742.

Guicciardini, N. 2009, *Isaac Newton on Mathematical Certainty and Method*, Cambridge MA, MIT Press.

Harcourt, G.C. 2003, "The Cambridge Economic Tradition", in King, J., *The Elgar Companion to Post Keynesian Economics*, Cheltenham, U.K. and Northampton MA, Elgar, 44-51.

Harcourt, G. 2006, *The Structure of Post-Keynesian Economics: the core contributions of the pioneers*, Cambridge, Cambridge University Press.

Hayek, F. A. 1948, *Individualism and Economic Order*, Chicago and London, Chicago University Press.

Jevons, W. S. 1888[1871], *Theory of Political Economy*, London and New York, Macmillan.

Keynes, J. M. 1973 [1921], *The collected writings of John Maynard Keynes, vol. VIII: A Treatise on Probability*, London. Royal Economic Society.

Keynes, J. M. 1936, *The General Theory of Employment, Interest and Money*, London, MacMillan.

Keynes, J. M. 1963, "Newton, the man", *in* G. Keynes (Ed.), *Essays in biography*, New York, W. W. Norton & Co, 310-323.

Lawson, T. 1985a, "Uncertainty and Economic Analysis", *Economic Journal*, 95, 909-927.

Lawson, T. 1985b, "Keynes, Prediction and Econometrics", *in* T. Lawson and H. Pesaran (eds.), *Keynes's Economics: methodological issues*, London, Croom Helm, 116-133.

Lawson, T. 2003a, *Reorienting Economics*, London, Routledge.

Lawson, T. 2003b, "Ontology and Feminist Theorising", *Feminist Economics*, 9, 119 - 150.

Leamer, E. 1983, "Let's take the con out of Econometrics", *American Economic Review*, 73, 31-43.

Marshall, A. 1890, *Principles of Economics*, London, Macmillan.

Nash, J. F. (1951), Non-Cooperative Games, *The Annals of Mathematics*, 2nd Ser., 54, 286-295.

Pasinetti, L. L. 2005, "The Cambridge school of Keynesian economics", *Cambridge Journal of Economics*, 29, 837–848.

Pratten, S. 1998, "Marshall on Tendencies, Equilibrium and the Statical Method", *History of Political Economy*, 30, 121-63.

Pigou, A. C. 1920, *The Economics of Welfare*, London, Macmillan.

Ricardo, D. 1821[1817], *On the Principles of Political Economy and Taxation,* London, John Murray, third edition.

Rosselli, A. (2005), "Sraffa and the Marshallian tradition", *European Journal of the History of Economic Thought*, 12, 403-423.

Runde, J. 1994, "Keynes after Ramsey: In Defence of 'A Treatise on Probability'", *Studies in History and Philosophy of Science*, 25, 97-121.

Schumpeter, J. 1994[1954], *History of Economic Analysis*, London: Routledge.

Sen, A. 2003, "Sraffa, Wittgenstein, and Gramsci", *Journal of Economic Literature*, 41, 1240-1255.

Shapiro, A. 1993, *Fits, Passions, and Paroxysms: Physics, Method, and Chemistry and Newton's Theories of Colored Bodies and Fits of Easy Reflection*, Cambridge University Press, Cambridge.

Sidgwick, H. 1883, *The Principles of Political Economy*, London, Macmillan.

Sraffa, P. 1925, "Sulle relazioni fra costo e quantita prodotta", *Annali di economia*, 2, 277–328.

Sraffa, P. 1926, “The laws of returns under competitive conditions”, *Economic Journal*, 36, 535–550.

Sraffa, P. 1960, *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*, Cambridge: Cambridge University Press.

Veblen, T. 1900, “The preconceptions of economic science: III”, *Quarterly Journal of Economics*, 14, 240 - 269.

Von Neumann, J., and O. Morgenstern 1944, *Theory of Games and Economic Behavior*, Princeton, Princeton University Press.

Walras, L. 1926[1874]. *Elements d'economie politique pure; ou, Theorie de la richesse sociale*, Paris, Pichon et Durand-Auzias; Lausanne, Rouge.

Whitaker, J., (ed.), 1975, *The Early Writings Economic Writings of Alfred Marshall* (2 vols.), London, Macmillan.

Whitehead, A.N. 1929, *Process and Reality. An Essay on Cosmology*, London, Macmillan.

Whitehead, A. N. 1938, *Modes of Thought*, Cambridge, Cambridge University Press.