Orthogonal Time in Euclidean 3-space

Being An Engineer's Attempt to Reveal
the Copernican Criticality of Alfred Marshall's
Historically-ignored 'Cardboard Model'

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Abstract

This paper begins by asking a simple question: Can a farmer own and fully utilize <u>precisely</u> five tractors **and** <u>precisely</u> six tractors at the same time? Of course not. He can own five ***or*** he can own six but he cannot own five ***and*** six at the same. The answer to this simple question eventually led this author to Alfred Marshall's historically-ignored linguistically-depicted 'cardboard model' where my goal was to construct a picture based on his written words. More precisely, in this paper our overall goal is to convert Marshall's ('three-dimensional') *words* into a three-dimensional *picture* so that the full import of his insight can be appreciated by all readers.

After a brief digression necessary to introduce the reader to the intricacies of Euclidean 3-space, plus a brief digression to illustrate the pictorial problem with extant theory, the paper turns to Marshall's historically-ignored words. Specifically, it slowly constructs a visual depiction of Marshall's 'cardboard model'. Unfortunately (for all purveyors of extant economic theory), this visual depiction suddenly opens the door to all manner of Copernican heresy. For example, it suddenly becomes obvious that we *can* join the *lowest* points on a firm's series of SRAC curves and thereby form its LRAC curve; it suddenly becomes obvious that the firm's series of SRAC curves only *appear* to intersect because mainstream theory has naively forced our three-dimensional economic reality into a two-dimensional economic sketch; and it suddenly becomes obvious that a two-dimensional sketch is analytically useless because the 'short run' (SR) never turns into the 'long run' (LR) *no matter how long we wait*.

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1. The Geometry of Euclidean 3-space

We start with Figure 1. It's a simple open-top cardboard box. Notice that we pretend we have X-ray vision so we can see through the cardboard, if required. Several things need to be noted:

1) One corner is labelled 'O' for origin because this will generally be our basic reference point.

2) Angles ZOX and AYB appear as right angles[1] because they *are* right angles *and* because they lie 'in' or 'parallel to' the plane of the paper.

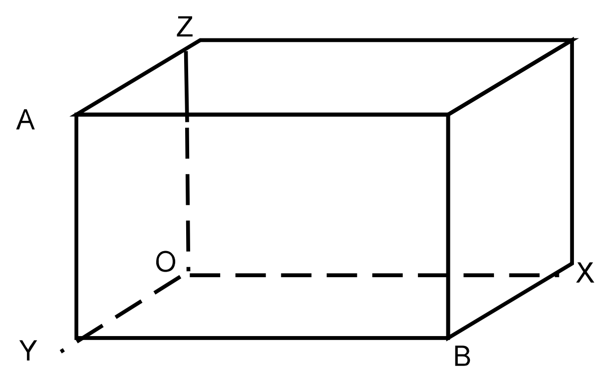
3) ***All*** other angles (e.g., angle AYO) are also 'right angles' but they do not *appear* to be right angles when a three-dimensional sketch is forced onto a two-dimensional page.

4) Thus Figure 1 is an orthogonal projection[2] of a simple cardboard box.

We focus our attention on the far lower-left corner. As mentioned above, this shall be the origin of our journey into Euclidean 3-space so we labelled it as Point O. Next, to aid in the visualization of what is before us, we imagine that the box has been pushed all the way back against a large piece of white paper thus Point Z, Point O and Point X will be touching the paper. In other words, the side identified as ZOX is a surface laying in the plane of our paper.

Now we look at the side identified by Point A, Point Y and Point B. These three points also create a plane surface but note that, even though AYB is also a plane surface it does not lay in the plane of our paper. It is a flat surface which is parallel to the plane of the paper (and to ZOX). This leaves us with a three-dimensional set of axes on which to place our various musings about reality, Figure 2A.

Fig. 1 A Simple Open-top Cardboard Box



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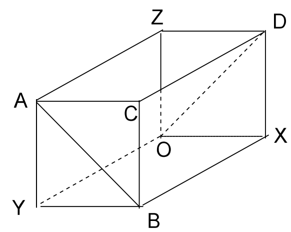
[1] The word 'right', as used in geometry, has nothing to do with 'correctness'. A 'right angle' is the angle formed by two intersecting straight lines when the angle between them is 90 degrees.

[2] Orthographic projection (or orthogonal projection) is a means of representing a three-dimensional object in two dimensions. It is a form of parallel projection, where all the projection lines are orthogonal to the projection plane, resulting in every plane of the scene appearing in affine transformation on the viewing surface. Orthographic projection - Wikipedia;

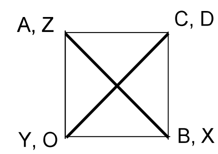
*https://**en.wikipedia.org**/wiki/Orthographic_projection*

Now we can start to hone in on the crux of the fundamental problem. In order to do this, we re-draw Figure 1 as viewed from a slightly different angle (Figure 2A), we change the proportions to aid in visual clarity and we add lines A-B and O-D. Notice that, in Figure 2A, these two lines do **not** appear to intersect when drawn in an orthogonal projection (i.e., when drawn in a picture which is closer to our 3-dimensional reality) yet, when we naively force the picture back into a 2-dimensional sketch (Figure 2B), *it now appears as if they do intersect*. Here's the reason: we have inadvertently cast a 'shadow' of Figure 2A back onto our paper (Figure 2B). ***Thus a researcher who was given <u>only</u> Figure 2B on which to base his/her analysis would probably assume that lines A-B and O-D intersect when, in reality, they do not.***

<u>Fig. 2A</u> 3-D Reality



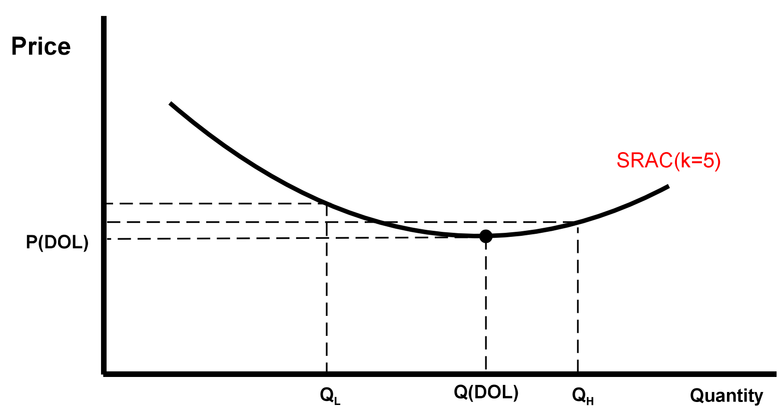
<u>Fig. 2B</u> The *apparent intersection* of two *non-intersecting* lines when a 3-dimensional reality is projected as a 'shadow' onto the (back) plane of the paper (i.e., a misleading 2-D sketch of 3-D reality).



2. Honing In: The Geometry of 'the Short Run' Sketched in Two Dimensions

Let us now let use what we have learned by applying it to an examination of the short run average cost (SRAC) curve for our farmer who owns *precisely* five output-producing tractors, Figure 3. Note that we have re-labelled the vertical axis as 'Price' and have re-labelled the horizontal axis as 'Quantity' so as to be consistent with conventional economic labelling. Note, also, that we will use either of two standard mathematical expressions to indicate our farmer's capital constraint. Specifically, we will express his ownership of tractors as SRAC(k=5) or even more simply as SRAC(5), depending on our needs at the moment. It is most important that the reader fully understands that, mathematically, the two expressions mean the exact same thing: our farmer - at the time of our initial examination of his 'physical capital' - owns precisely

<u>Fig. 3</u> A Typical 'Short Run' Average Cost (SRAC) Curve



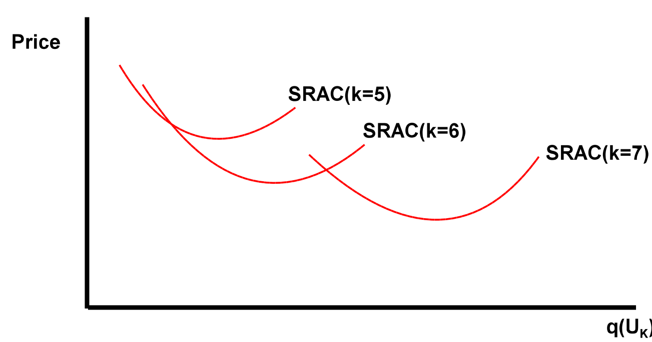
five usable output-producing tractors. As discussed in a just moment, we will let him (if he wishes) add to his physical capital by allowing him the option of purchasing an additional tractor(s) next year (or 'whenever').

In the meantime, as mentioned above, we have recast everything in a format more suitable for an economic analysis of 'the short run'. Also note that we have identified the lowest point on the farmer's SRAC curve as Q(DOL). This is the farmer's Design Output Level when his tractors are being utilized at 100%, no more, no less. This is the production level where the farmer's short run average costs are at a minimum when he owns five tractors.

Three additional points need to be mentioned here and we put the crucial point first. Figure 3 is a picture of reality. It is not dependent on any economic theories; neither is it dependent on any (relevant) 'simplifying assumptions'. Second, Figure 3 (for any particular real-world firm) would be constructed from collectable and/or calculatable real-world data thus Figure 3 is a visual presentation of the minimum selling prices (for various levels of output) which would be financially acceptable to the firm for some sustainable[3] future, given its particular and *extant* arrangement of capital and labor, *ceteris paribus* (loosely translated: 'all other things held constant'). Third, we shall not, at this juncture, allow quibbling over the components of 'production cost'; we let the reader select his/her own components and require only that rigorous consistency be maintained throughout.

Moving on, Figure 4, we let there be a correctly-anticipated increase in business and therefore allow our farmer to contemplate an increase in his capital; specifically, he contemplates buying one *additional* tractor. [Note that we now include SRAC (k=6) in Figure 4.]

Fig. 4



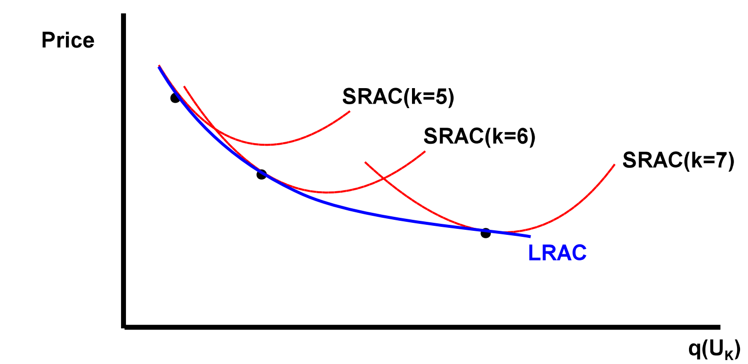
Before we proceed further, it's important to understand that, in this paper, our analytic requirements are rather strict. First, the new tractor is **not** a 'replacement' tractor for one that is not fixable (it's an *additional* tractor). Second, it has **exactly** the same performance capabilities as the rest of our farmer's tractors (same horsepower, etc., etc.). Third, it does **not** have any technical

3 Notice that we carefully avoided any insinuations regarding 'short run' and/or 'long run'.

improvements (e.g., if the original tractor had a carburettor, this one has a carburettor, not fuel-injection). [4]

Now we can move on. We let our farmer also contemplate the purchase of two additional tractors (again, Figure 4), thus increasing the number of fully-utilized tractors to seven. When the resulting SRAC curve for the seventh tractor is added to our figure and we force everything into a two-dimensional sketch, we begin to see the problem more clearly, Figure 5. A two-dimensional sketch of our three-dimensional reality gives the viewer the completely erroneous impression that the various SRAC curves intersect in various places and, to the best of this author's knowledge, this is the current state of affairs regarding extant economics theory's current visualization of a firm's SRAC curves. More importantly, when viewed as in Figure 5, we are forced into the standard 'tangency solution' when we try to construct the firm's LRAC curve because, while a firm can have short-run *economic* losses and long-run business profits at the same time, it cannot have short-run *business* losses and long-run business profits at the same time.[5] Now can we can turn to Marshall's 'cardboard model' and see how he thought the mis-perception problem should be solved.

Fig. 5 Extant economics' simple but misleading presentation of the relationship between the firm's series of SRAC curves and its LRAC curve.



Remember: A firm can have short-run *economic* losses and long-run *business* profits but it cannot have short-run *business* losses and long-run *business* profits. Thus the 'tangency requirement' in this too-simple sketch.

4

In subsequent papers we will be much more lenient because we will want to start moving much closer to reality. Specifically, realistic leniency will allow us to push well beyond Marshall and thus examine our farmer's options in Euclidean 5-space.

5

It took this author a long time to fully grasp the crucial difference between ***economic*** profits and ***business*** profits. An (external), i.e., real-world lack of adequate competition determines the size of the firm's economic profits whereas a lack of (internal) business acumen determines the size of the firm's business profits. Confusion can arise because both are calculated based on 'left-over' money.

3. We Begin in Ernest: Marshall's Obscure Footnote

We begin by examining the first part of Marshall's footnote. It explains how we could come much closer to our economic reality with regards to this particular economic sketch.

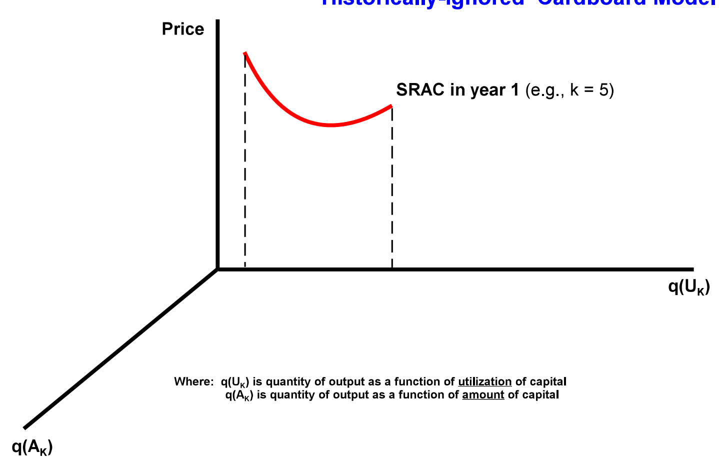
> 'We could get much nearer to nature if we allowed ourselves a more complex illustration. We might take a series of curves, of which the first allowed for the economies likely to be introduced as a result of each increase in the scale of production during one year, a second curve doing the same for two years, a third for three years, and so on.' [6]

Obviously Marshall is not yet describing the precise same picture that we are herein considering but, already, he clearly recognized the need to go beyond the standard two-dimensional schema when trying to visualize the interactions between *three* economic variables. Now let us turn to the last half of his footnote.

> 'Cutting them out of cardboard and standing them up side by side, we should obtain a *surface*, of which the three dimensions represented amount, price and time, respectively.' [7]

Notice that Marshall used the words '... amount, price and time...'. We chose to avoid the actual use of the word 'time' because the pictorial location of any particular SRAC curve does not depend on the passage of time, *per se*; it depends, instead, on the firm's state of production affairs at the end of any 'time interval' during which capital was increased. In other words, our farmer might buy additional tractor(s) at the end of one year

Fig. 6A How to Construct Alfred Marshall's Historically-ignored 'Cardboard Model'



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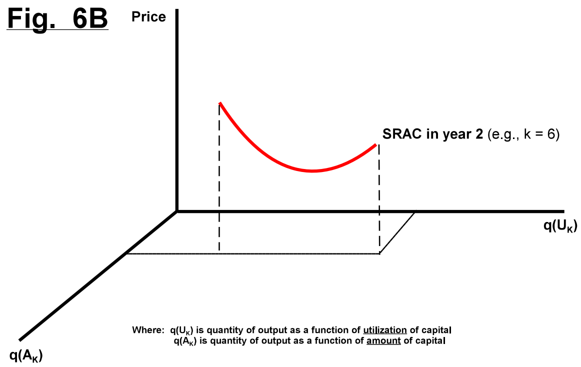
[6] Marshall, Alfred, *Principles of Economics*, 8th ed., Porcupine Press, Philadelphia, 1990, App. H, Art. 3, footnote 2, p. 667.

[7] Marshall, 1990 (italics in original).

or he might buy it/them at the end of two years or at the end of three years. The important point is that our farmer increases his output-producing capital in ‘clumps’ (he cannot utilize ½ of a tractor).[8] Anyway - usurping some poetic licence regarding Marshall’s precise words - we illustrate an unambiguous visual depiction of our farmer’s initial situation (i.e., k=5), Figure 6A.

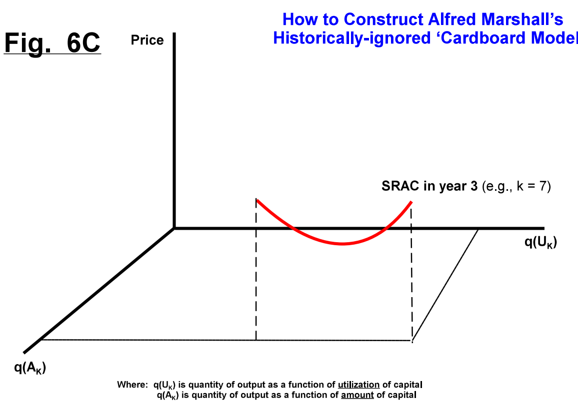
Next, we let him contemplate the purchase of one additional tractor thus he would then own six tractors, Figure 6B. Note that, in figures 6A, 6B and 6C, the axis coming ‘out’ of the page has now been re-labelled as $q(A_k)$; i.e., output is shown as a function of the amount of capital, not as a function of time.

Fig. 6B



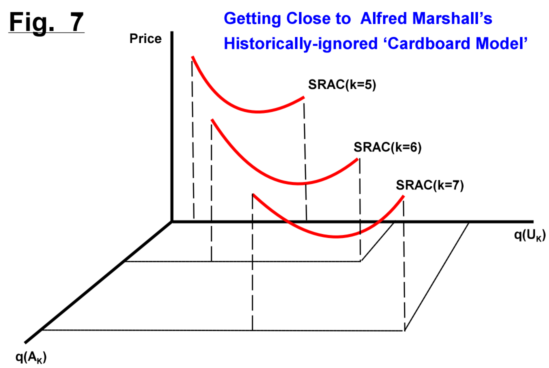
Finally, we let him consider adding two tractors at the end of the first year, Figure 6C. Certainly, he could have chosen to buy no additional tractors (k=5); he could have chosen to buy one additional tractor (k=6) or he could have chosen to buy two additional tractors (k=7). The wisdom of his decision regarding (a) how many additional tractors to contemplate buying (if any) and (b) when to buy them would, of course, be almost totally dependent on him having reliable real-world cost data and/or cost estimates.

Fig. 6C



Now we can combine figures 6A, 6B and 6C so as to form Figure 7, thus coming very close to reaching Marshall’s cardboard model.

Fig. 7

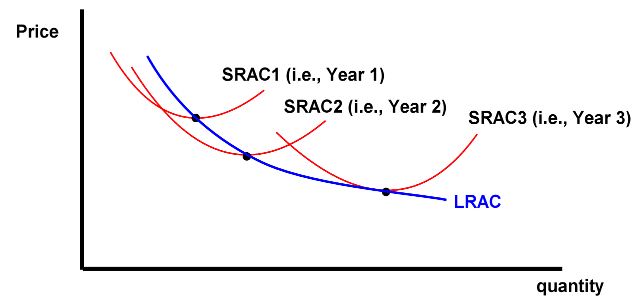


[8] ‘clumps’ might suggest that a ‘quantum economics’ approach be considered but unfortunately, that terminology is already gaining unwarranted currency.

But, before we take the last step, it seems important to show that - if we wanted to - we could (confusingly) force Marshall's 3-D model back into a 2-D sketch, Figure 8. Note the subtle but crucial difference between Marshall (Figure 8) and extant theory (Figure 5). Specifically, Marshall's depiction allows for a 'low point solution' to the SRAC vs LRAC problem whereas extant theory requires the 'tangency solution'.

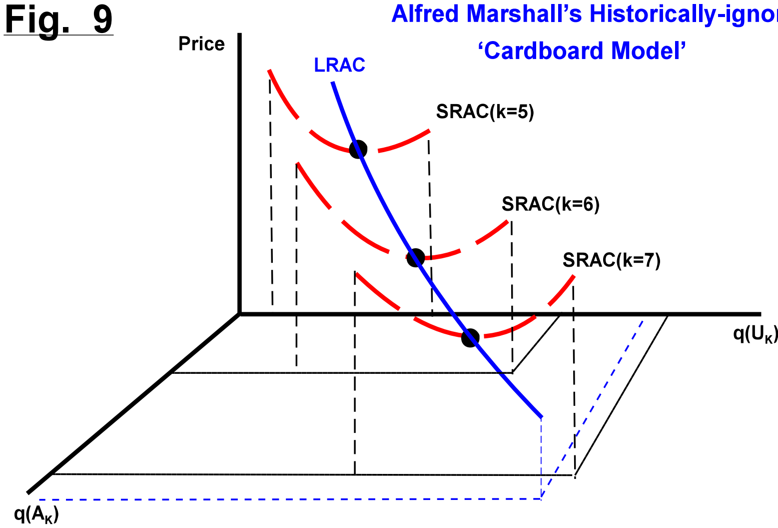
Fig. 8 A Misleading Simplification of Marshall's Cardboard Model

We could, if we wanted to, force Marshall's 'Cardboard Model' back into a 2-dimensional sketch. But... it would not illustrate all the crucial details provided by his 3-dimensional 'sketch'.



Now let us take the last step. Let us view Alfred Marshall's three-dimensional 'cardboard' model as this engineer believes it was meant to be viewed, Figure 9. In Figure 9, we show the basic 'ribs' which form the skeleton of Marshall's short-run vs long-run 'surface'. And, given that a clear appreciation of the SRAC vs LRAC arrangement seems a necessary precursor to more advanced economic theorizing, it would seem that it is time for Marshall's *three-dimensional historically-ignored* 'cardboard model' to be given its rightful place as one of the several 'foundations' of modern economic theory.

Fig. 9 **Alfred Marshall's Historically-ignored 'Cardboard Model'**



Where: $q(U_K)$ is quantity of output as a function of utilization of capital (for any particular amount of capital)
$q(A_K)$ is quantity of output as a function of amount of capital (for any particular utilization of capital)

4. Conclusions

It should now be obvious that the distinction between the 'short run' and the 'long run' has absolutely nothing to do with calendar or clock. Indeed, the distinction must be based solely on the various sizes of the 'clumps' of output-producing capital that a representative firm actually has available at any given instant. Basically, our farmer chooses to own a certain number of tractors (i.e., he chooses a particular short-run curve from a set of long-run options) for his 'course tuning' of output capability then 'fine tunes' his actual output - **while 'stuck' on that pre-selected SRAC curve** - so as to maximize his profits in accordance with market demand. All things considered, we arrive at the following conclusions:

1) We *can* join the *lowest* points on a firm's series of SRAC curves and thereby form its LRAC curve;
2) The firm's series of SRAC curves only *appear* to intersect because mainstream theory unnecessarily (and misleadingly) forces our *three-dimensional* economic reality into a *two-dimensional* economic sketch;
3) A two-dimensional sketch is analytically useless because the 'short run' (SR) never turns into the 'long run' (LR) *no matter how long we wait.*

In summary - when the words of Alfred Marshall are recognized as being a set of instructions and we then draw a picture based on those words - we begin to understand that (using modern engineering terminology) 'the short run' and 'the long run' are orthogonal functions in Euclidean 3-space.[9]

[9] Those readers already familiar with orthogonal functions probably realize that, while the axes (price, quantity, capital) *are* orthogonal, a real-world firm's LRAC curve will almost never be fully orthogonal to its collection of SRAC curves because the firm's LRAC curve is actually a 'directional derivative', not a true 'partial derivative' of the overall production function. Our purpose herein was to bring modern attention to Marshall's historically-ignored 'cardboard model' thus we used relatively simple illustrations and/or words and leave gradients and vector calculus to the 'quants'.