The Cambridge Controversy. A note

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Abstract

A debate, which started at the late 1950's and lasted up to the early 1970's, and is known as the Cambridge controversy, is the subject of this paper. The core of that debate was the composition of capital and moreover the production and the reproduction of commodities. The "roots" of the Cambridge Controversy can be found in the literature of K. Marx and D. Ricardo. However both schools of thought have underestimated the important role of price normalization, leaving the problem of choosing techniques unexplained.

Keywords: Surrogate Production Function, Cambridge Controversy, Price normalization, reswitching, Capital formation, Capital composition.

JEL: B24, B25, B51

The neoclassical perspective

The neoclassical point of view was expressed primarily by P. Samuelson and R. Solow.

In P. Samuelson (1953) the properties of the neoclassical production function (NPF hereafter) were presented. The NPF existed (and still does) in a perfect competition framework. A given homogeneous function of inputs and n producible goods existed[1]. Production factors obeyed in the "Law of Diminishing Returns" (P. Samuelson, 1953), which as he believed could be empirically confirmed. For him the existence of perfect competition implied

[1] In other words there was a given proportion of input (production factors)-outputs

that marginal productivity value (i.e. the price of the marginal product of every factor) would be common in all markets.

Later in P. Samuelson (1962) he tried to defend the Aggregate Production Function as expressed in R. Sollow (1957). He assumed that the entire economy produced a homogeneous output, with Constant Returns to Scale (COS), using a non-producible production factor (labour) and capital goods. This production pattern was described by the Surrogate Production Function (SPF hereafter). Moreover he introduced a relation between labour (nominal wage) and profits (profit rate), which he called, while trying to define steady state positions, Factor-Price Frontier[2]. The points placed in the upper envelope represented steady states of the system. Consequently, for a given profit rate the nominal wage could be determined. Then the rent for using the means of production, such as machines, labor, land etc., the production price of goods (final product) and all other endogenous variables of the systems were being determined. In addition, he considered the case of the presence of heterogeneous capital. Under the special assumption of his model the occurred w-r relation was linear regardless if one or more commodities were used as means of production. Each of these commodities used in a given proportion alongside with homogenous labour and produced a given commodity or basket of commodities[3]. The linear w-r implied that without substitution, in a steady state, a given proportion of output and homogenous labor is produced[4]. The latter is what P. Samuelson defined as "«Exact model of the Clark-Ramsey Parable". In this model a homogenous net output is produced, using jointly a "jelly of capital goods" and homogenous labor. This output consists of capital goods and net capital. In fact, this function is

[2] The latter is also known as the "w-r relation".

[3] In the special case that P. Samuelson considers the production processes work independently. On the other hand, these production processes are represented by a diagonal coefficient Matrix -in linear algebra terms- or with a matrix with linear dependent rows or columns.

[4] According to P. Samuelson (1962) in the special case of a finite number of commodities, used as means of production, the w-r curve did not have the familiar shape of an Isoelastic Production Capabilities Curve (PCC hereafter) but was a -L- Shaped PCC instead.

characterized by Constant Returns to Scale (COS). In the same time this function was compatible with the marginal changes of production factors[5].

P. Samuelson called the aforesaid capital "Jelly" "Surrogate (Homogeneous) Capital". The presence of "Surrogate (Homogeneous) Capital" (J) implied a linear w-r curve, with stable proportions of net output and homogenous labor[6]. So after the production factors prices were being determined then the production prices of capital goods[7] and other goods were been determined too. Consequently, the product of the price of a given quantity of capital with the respective price determined the "Surrogate (Homogeneous) Capital" (J)

(1) $$J = p_a K_a + p_b K_b + ...$$

, p_i represents the price of a quantity of heterogonous capital i and K_i the quantity of capital i.

D. Levhari (1965) believed that it was possible for the reswitching[8] phenomenon appear in a single industry, but not in the entire economy. More specifically in the special case of a non-decomposable single production

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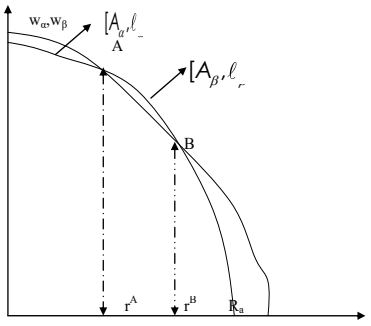
[5] In other words, it could be a Cobb- Douglas function, such as $Y = K^a L^{1-a}$, which could be - as Samuelson admits- compatible with the statistical "maneuvers" of R. Sollow.

[6] This proportion coincides with the term of labor productivity

[7] These capital goods represent physical quantities of capital

[8] For the unfamiliar with linear systems of production reader, as reswitching is being defined the phenomenon that the system returns to a production technique, which has earlier excluded as not so profitable. In other words, according to the reswitching phenomenon it is absolute possible into two different profit rate intervals to be chosen the same technique without the aforesaid intervals to be continuous. This situation could be described in the hereunder scheme:

Let two different production techniques exist, $[A^{(\cdot)}, \ell$ ι, b Moreover let w and r represent the nominal wage and the profit rate respectively



technique he introduced an algorithm, according to which for a given profit rate, the system converged, in cost minimization criterion terms, to the use of one and only technique. This choice of technique was independent of which technique's prices would be set as initial.

Summing up the neoclassical point of view of the 70 s, it can be said that: a) there was an inverse relation between profit rate and capital intensity. In other words they believed that when the profit rate increases firms tend to use more labour instead of capital (in the case that capital and labour were fully substituted production factors)[9]. b) In a perfect competition framework, the allocation of income could be explained in marginal productivity terms. In General Equilibrium Models demand and supply were what determined not only the production factors price but the price of goods as well. In other words demand and supply determined all income types (wages, profits, land rentals etc). c) Capital could be measured and be summed up to a homogenous production factor. In that way capital could be determined within a General Equilibrium Model framework.

As we have seen above for the neoclassical economists a crucial -for the validity of their theory- distinction between goods, capital good and labour was made. This distinction had led to a marginal productivity based, -or in other words supply demand-based-, determination of income from using production factors. Next the price of consumption goods could easily be determined too. It was clear for them that all prices of goods and types of income could be easily determined based only in technological criteria, which in fact set the behavour of the market, which next determined the price of goods and allocated income.

[9] It is also trivial that when the nominal wage increases firms tend to use more capital instead of labour.

The neoricardian perspective

The neo-ricardian perspective on the other hand was expressed primarily by P. Sraffa, J. Robinson, and P. Garegnani.

In contradiction to what neoclassical economists believed, capital could not be measured and summed to a capital "Jelly" according to the neoricardians. Production techniques were linear with uniform profit rate and nominal wages. Within this analytical framework they studied the relation between profit rate, wages and price determination systems. For them prices could be determined based not only to technological attributes of the system but to income distribution variables as well. The commodities were divided into non-reproducable –the price of which was determined by scarcity- such as Labour and into reproducable commodities, which served the dual role of mean of consumption and mean of production –or in Sraffain words commodities produced by means of commodities-. They found that the marginal productivity theory was not valid and the corresponding SPF could not exist under more realistic assumptions. As G. Harcourt (1972) had pointed out they believed that they have shaken down the SPF and the entire structure of supply and demand curves for Labour and means of production. That way the marginal productivity and theory of value had been shaken down and the economic theory had been moved to a valid –a la Ricardo- place.

Neoricardian economists have spotted phenomena such as reswitching and capital reversing[10] which had weakened the marginal productivity theory.

[10] It is important to state here that these phenomena were first noticed by pioneer economists such as Joan Robinson (1953-4), D. Champernowne (1953-4) and P. Sraffa(1960) . As capital reversing is considered the situation, that the capital value has a monotonic behavior in profit rate terms, such that less capital intended techniques are being linked with a lower profit rate. In other words it is possible for a less automatized technique to be linked with a higher profit rate. This reverse relation was named by Joan Robinson as "perverse" relation Capital Reversing and reswitching phenomena ensured the possibity that the same quantity of heterogeneous capital could have more than one prices for the same profit rate.

Joan Robinson (1953-54) studied the "nature" of capital into the SPF, trying in the same time to a) to decode the forces that affected the capital's and labor's growth rate and economy as a whole b) find how the technology evolution affected capital circulation, income distribution and choice of techniques, which in fact determined the investment behavior. According to her with given labor and capital reserves and with a given substitution of inputs, the income distribution in workers and capitalists could be delivered. For her it was impossible, even in a non-growing economy, It was impossible according to her, even in a non-growing economy, for the rate of profit to be determined based only in technological criteria. For that conclusion she constructed an a la Sraffa model, which produced a single output, using re-producible capital and labor. It was impossible for her to find a unit of measure (UoM)[11], based on which and independently of the income distribution, the social or total capital could be summoned in a single number.

Levhari's algorithm[12] had been the motive for P. Garegnani(1966) for a critique against the SPF and neoclassical theory. While studying the w-r relation, and given the reverse relation of profit rate and capital intensity in neoclassical models[13] -which as we have seen above made the unambiguous choice of techniques possible and the reswitching[14] phenomenon impossible- he proved the existence of reswitching. The latter drives the neoclassical theory in absurd (P. Garegnani 1970), since "the system" in the case of reswitching returned to a technique which was earlier abandoned since it were in the same time both less profitable and less capital intended.

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[11] D. G. Champernowne (1953) accepted J. Robinson aspect, regarding the impossibility of measuring capital, since he failed finding such an UoM. However he questioned the possibility for the same physical quantity of capital to have a different price in different steady states.

[12] Regarding the validity of the Levhari's algorithm in Joint Production Techniques, the concerned reader could find C. Bidard(1990) very interesting.

[13] In other words as the capital intensity grows with a falling profit rate.

[14] The capability of (re)switching techniques was already been proved in L. Pasinetti (1966), P. Garegnani(1966), J. Robinson and K. A. Naqvi (1967) and of course in P.Sraffa (1960).

For L. Passinetti(1966) there was no functional relation between capital intensity and profit rate. Based on the well-known P. Sraffa's (1960) numerical example, he also tried to show the faults in the theory of D. Levhari, P. Samuelson and R. Sollow. He proved -that in that numerical example terms- it is absolute possible for the reswitching to exist in the entire economy, and not in a single industry only. In addition he recognized the weakness of neoricardian models regarding the existence of a second primary input -or fixed capital in other words-. In this case he indirectly proposes what in the last chapter of P. Sraffa (1960) is stated, i.e. the existence of fixed capital converts a single production technique in a joint production one[15]. For P.Sraffa(1960) the conclusions of his analysis seem to remain unaffected in a joint production framework.

K. Bharadwaj(1970) not only accepted the existence of the reswitching phenomenon, but tried to determine the maximum number of (re)switch points in single production. It was proved that the maximum number (re)switch points in 2 non-decomposable neighboring techniques producing n basic products were equal to n.

In the case of 2 decomposable neighboring techniques showed that a) the maximum number (re)switch points is equal to the number of commodities that enter directly or indirectly to the production processes of all commodities, i.e. equal to the number of the basic products and b) in the case that the 2 decomposable neighboring techniques differ in the production process of one non-basic commodity the maximum number (re)switch points is equal to the number of the basic products plus the number of the non-basic products that enter directly or indirectly in the production process of the aforesaid non basic product -that the two neighboring techniques differ-.

[15] B. Schefold(1978), Salvadori and Kurz(1994),G. Stamatis (1995), C.Bidard (1999) tried to find out how the existence of joint production affects the capital intensity - profit rate relation.

Another perspective. A note

Opposite to what both neoricardian and neoclassical economists believed, choice of techniques does not depend on the technological attributes of the system and (or) income distribution, or entirely on technological attributes only respectively, but we believe that price normalization plays an important role in the choice of techniques (as G. Stamatis (1993) had shown).

For neoclassical economists w-r curve was always linear, since the (w/r) ratio was always constant and independent of prices. In the special case that P. Samuelson (1962) described the n production processes –for the n produced goods- were independent from each other, since the corresponding A (nxn material input matrix) was diagonal[16]. Moreover the existence of a diagonal A, it was sufficient to show that neither the number nor the place of the switch points change with price normalization. The linearity of w-r ensured the exclusion of capital reversing and reswitching phenomena. It ensured also the existence of a monotonic falling relation of capital intensity –namely the ratio of capital Jelly (J) and total labour (L)-and profit rate (r)-.

The neoricardians on the other hand had shown that within a heterogeneous capital framework, the w-r relation was non-linear and therefore it was absolute possible for the reswitching and capital reversing phenomena to exist. P. Sraffa mostly had shown that the existence of a linear w-r curve was only a special case. He had shown that the introduction of a standard commodity, such as the Sraffaian one, in which net output and real wages shared the same composition, ensured the existence of a linear w-r curve , and therefore an unambiguous choice of techniques.

Nevertheless both neoricardians and neoclassicals had underestimated –and still do- the important role of price normalization. It has been shown in G. Stamatis (2001) and K. Manoudakis(2010) that the price normalization

[16] The existence of a diagonal A matrix ensured that net product and material inputs shared the same composition.

changes the number and the place of (re)switch points. It also changes the shape and the slope of the w-r curve. Also, the ordering of techniques –especially for the ones under the upper envelope- changes with price normalization. The reason for all these changes is quite simple. When we introduce a price normalization equation we do not choose among techniques but among normal subsystems instead (G. Stamatis (1997a)). These discrete normal subsystems have different net output and real wage composition and consequently lead to a different income distribution and moreover to different production prices. The only cases that we can really choose and order techniques among others –and not normal subsystems- are the corn economies and the Charassoffian (G. Stamatis (1999)) systems of production (G. Stamatis 2001, K. Manoudakis (2010)). Moreover we can consider an a la von Neumann, system of production as a special case of a charassofian system for the case of joint production. In all these changes, since inputs and outputs share the same composition it is absolute possible for profit to be determined regardless the production prices and price normalization.

More precisely let us see how price normalization affects the findings of the Cambridge controversy. In trivial linear systems theory it is well-known that capital intensity, profit rate and nominal wage are determined by the price determination equation system. Solving this set of equations we find that (K. Manoudakis 2010): a)Capital intensity of the normalized subsystem is a falling function of the profit rate b)Capital intensity of the normalized subsystem depends on the standard commodity. c)The slope of the w-r curve is equal to the capital intensity of the normalized subsystem and therefore depends on price normalization d)The position of the w-r curve is equal to the rate of the net product of the normalized subsystem in price terms and the respective quantity of direct labour in the normalized subsystem e)The shape (convex, or non-convex or linear[17]) of the w-r curve also depends on price

[17] In the case that the w-r curve is linear, the capital intensity is constant. The unambiguous choice of techniques is certainly unambiguous. A linear w-r relation is possible when Matrix A can be diagonalized or when prices are normalized with Sraffaian (equal to positive right eigenvector of A,

normalization, since the profit rate is a function of the normalization vector f)Also the quantity and the composition of the capital used in producing a given net product, such as the standard commodity, may change if the specific product changes too.

On the other hand in Charassofian systems of production –and of course in an a la von Neumann and in corn economies the composition of capital is independent on price normalization. More precisely in charassofian systems and in a la von Neumann economies inputs and outputs (the coefficient matrix includes also the real wage) have the same composition. Therefore the profit rate is independent of prices and independent of price normalization, and is determined only based in technological criteria, as ratio of the net product and the used means of production.

In corn economies the profit rate is determined independently of prices since it is determined only based in technological criteria, as ratio of the net product and the used means of production.

Concluding Remarks

It is true that the Cambridge Controversy revealed the scientific and ideological differences of the two Schools of Economic Thought. The latter was recognized by J. Stiglitz (1974). The Cambridge Controversy emphasized the role of the production factor "Capital" in economic theory but underestimated the significance of existence of the normalization equation and the corresponding normal subsystems. It is clear that the findings of that debate would be very different if the two schools took into account the standard commodity.

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corresponding to the maximum eigenvalue of A) or any other standard commodity such as the Miayo's one.

References

1. Bharadwaj, K. (1970). On the maximum number of switches between two production systems.» Schweizerische Zeitschrift Fur Volkswirtschaft und Statistik, vol. 106, pp. 409-429
2. Bidard, C. (1990) An algorithmic theory of the choice of techniques», Econometrica, vol.85, pp. 839-859
3. Bidard, C. (1999), Fixed capital and internal rate of return, Journal of Mathematical Economics 31, 523–541
4. Champernowne D. G. (1953) The Production Function and the Theory of Capital: A Comment, The Review of Economic Studies, Vol. 21, No. 2 , pp. 12-135
5. Garegnani, P. (1966), Switching of Techniques , Quarterly Journal of Economics
6. Garegnani, P. (1970). Heterogenous capital the production function and the theory of distribution», Review of Economic Studies, vol. 37, pp. 407-436
7. Harcourt G. (1970). Some Cambridge Controversies in the Theory of Capital, CambridgeUniversity Press.
8. Kurz, H/ Salvadori N. (1995). Theory of production, Cambridge University Press
9. Manoudakis K (2010), Choice of Techniques according to their profitability (in Greek), Panteion University of Social and Political Sciences, Athens
10. Pasinetti, L. (1966). Changes in the Rate of profit and switches of techniques, Quarterly Journal of Economics, vol.80, pp. 503-517.
11. Pasinetti, L. (1977), Lectures on the theory of production, MacMillan.
12. Robinson, Joan (1953-54). The production function and the theory of capital, Review of Economic Studies 21: 81-106.
13. Robinson J./ Naqvi K.(1967), The Badly Behaved Production Function, The Quarterly Journal of Economics, Vol. 81, No. 4, pp. 579-591

14. Samuelson P. (1966), A Summing Up, The Quarterly Journal of Economics, Vol. 80, No. 4, pp. 568-583 Published by: The MIT Press
15. Samuelson, P.(1962) Parable and realism in capital theory: The surrogate production function, Review of Economic Studies, vol.29, pp. 139-206.
16. Schefold, B. (1978α). Multiple product techniques with properties of single product systems Journal of Economics, vol. 38, pp. 29-53.
17. Solow, R.(1956) The Production Function and the Theory of Capital, The Review of Economic Studies, Vol. 23, No. 2 pp. 101-108
18. Solow, R.(1956) A Contribution to the Theory of Economic Growth , Quarterly Journal of Economics (Feb. 1956), Vol. 70, No. 1., pp. 65-94.
19. Solow, R.(1957) Technical Change and the Aggregate Production Function,The Review of Economics and Statistics, Vol. 39, No. 3, pp. 312-320
20. Stamatis, G. (1993), The Impossibility of a Comparison of Techniques and of the Ascertainment of a Reswitching Phenomenon, Jahrbuecher fuer Nationaloeconomie und Statistik, Band 211, Nr. 5-6, May, pp. 426-446
21. Stamatis, G. (1995), On the 'normal' behavior of indecomposable and multiply decomposable joint production systems, Issues in Political Economy (in Greek), No. 17, Autumn, pp. 73-134
22. Stamatis, G. (1997a), On the unambiguous ordering of linear production techniques with respect to their profitability or their 'cheapness', Issues in Political Economy (in Greek), No. 20, Spring, pp. 121-132.
23. Stamatis, G.(2001), Why the Comparison and Ordering of Techniques is Impossible, Issue of Political Economy 9, p.p. 5 - 44
24. Stamatis, G. (1999), Georg Charasoff. A pioneer in the Theory of Linear Production Systems, Economic Systems Research, vol. 11, pp. 15-30
25. Stiglitz J., The Cambridge Controversy in the Theory of Capital: A View from New Heaven, Journal of Political Economy,4, pp. 883-903,