In defense of a non-newtonian economic analysis through an accounting paradigm

Diana Andrada Filip* & Cyrille Piatecki†

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Abstract

The double-entry bookkeeping promoted by Luca Pacioli in the fifteenth century could be considered a strong argument in behalf of the multiplicative calculus which can be developed from the Grossman and Katz non-newtonian calculus concept provided that one goes from an additive bookkeeping system to a multiplicative one. In order to emphasize this statement, we present a brief history of the accountancy in its early time and we make the point of Ellermans research concerning the double-entry bookkeeping. The most astonishing point linked to this subject is to realize that not only the calculus, but also the accounting systems, have been the subjects of path dependency.

Keywords: Path dependency, Bookkeeping, Non-newtonian analysis
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*Babeş-Bolyai University of Cluj-Napoca, Romania and LEO, CNRS UMR 7322, Orléans University, France — diana.filip@econ.ubbcluj.ro

†LEO, CNRS UMR 7322, Orléans University, France — cyrille.piatecki@univ-orleans.fr

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1 Introduction

According to Kuhn (1962), a paradigm shift is a radical change of our way to understand the world such that no one inside the field of reflection where it occurs can ever refers to the corpus that was pregnant before the shift. But, for a paradigm shift to occur, the failure of the older paradigm must be acknowledged by a scientific community. And to acknowledge that there are failures, a scientific community must not be locked-in a way to look at some phenomenons or in a way to analyse those phenomenons.

And in complete opposition to what some could think, it is through our model of the world that we try to understand it because nobody has never been able to collect data without a conception of what are the data.

Now, because our cerebral enginery has evolved in such a way that, at birth, we are only able to conceive elementary additions and substractions, all other operations being the object of a complex process of learning — see for exemple Lakoff & Núñez (2000) —, we are locked in an additive conception of our world which is rarely questioned.

All could have changed in the beginning of the seventies, Grossman & Katz (1972) brings out a challenging new concept which define the derivative not as a a difference but as a ratio which fit exactly with our conception of growth. That is to say that they have proposed to replace the definition of the derivative from

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{to} \quad f^{\star}(x) = \lim_{h \to 0} \left(\frac{f(x+h)}{f(x)} \right)^{\frac{1}{h}}$$

Unfortunately, there is a gangway to go from newtonian to non-newtonian derivative. So if for instance a non-newtonian analysis of economic growth should be implemented, to obtain some refutable new results, the involved balance equation must also be non-newtonian, that is nonadditive. But is such an accounting system possible? As we will try to demonstrate in the following sections, this is a perfectly achievable goal that justify the approach developed by Filip & Piatecki (2014). Unfortunately, this goal would certainly never fulfilled since the accounting system is a locked-in.

2 The accounting locked-in

It is hard to imagined that the way we take record of our transactions could have been managed in a different way, better adapted to the description of the growth or decline of ours operations.

2.1 A brief accountancy history

It is acknowledge that human beings count since its emergence on this world. But a serious accountancy system like the double entry bookkeeping appears only in the 14th century Italy and not in the merchant civilisation of the middle east or in Greece or in Rome. According to Littleton (1933), there are 7 *key ingredients* which led to the creation of a double entry book keeping :

① *Existence of a private property system* : This is a mandatory ingredient because bookkeeping is concerned with the record properties, of transfert of properties and on property rights.

② *Accumulation of capital* : the growth of the human activity has been effective only when those with a know-how has been able to borrow ressources to those who possessed them in such a way that commerce and credit ceased to be trivial.

③ *Commerce at a widespread level* : At a local level, small volume trading does not create a pressure to organise a strong system of bookkeeping, because a simple accountancy is perfectly sufficient for small quantities transactions.

④ *Existence inter-personal credit with an enforcement guarantee* : If all transactions are untied on the spot, there is no incentive to keep any record. The enforcement guarantee need as a precondition the existence of an authority strong enough to sanction the non-payment of the principal and the interest due to the loaners. As is shown by the early civil code as the Ur-Numu or the Hamuraby ones — nearly 2285 B.C. —, the pre-existence of the State, if not a mandatory condition is necessary to the realisation of this ingredient.

⑤ *Writting* : Obviously, if one cannot write, because human memory is too fallible, there is no way to record any thing. The oral transfert of informations from people to people cannot offer any garanties of authenticity.

⑥ *Money* : It is also a mandatory condition, because without a *common denominator*, bookkeeping is nearly impossible. In the contrary, with money and from the point of view of bookkeeping, transactions are no more that a set of monetary values.

⑦ *An arithmetic* : This is also mandatory because without the mastery of an arithmetic there is no way to compute the monetary details of the transactions.

As signaled by Alexander (2002), many of these factors where present long before the 14th century but either there where not present in the same time and place or they where present in an unstructured form or with a not strong enough pregnancy. Therefore, in what concern writing, if it is not a necessary condition for civilization — the gallic civilisation is acknowledge as a civilization even if it never acquired writing —, it was mandatory to begin historical times since History could not exist without records. But even if the beginnings of arithmetic are contemporary of the civilization, arithmetic understood as the systematic manipulation of numbers, was not a tool acquired on a suffisant scale until the middle-age to help to the development of a double entry bookkeeping.

More than that, the nearly universal use of roman numeral in the occidental world long after that the arabic numeral have been introduced, has been a severe restraining factor because of the non existence of the zero in the roman numeral systems. Yet, a neutral element, and zero is the neutral element of the addition, is mandatory to the development of a double entry bookkeeping as would be shown in the section 2.2.

Nevertheless, in the ancient times, the problems encountered by mercantile or even States where alike with ours. For instance, because of tax collecting, governments had strong incentives to keep records of receipts and expenditures. As rich people often hired agents or used exclaves to perform their operations, they needed to realize audits to verify the honesty and/or skill of their factotum. But the illiteracy and the cost of writing — ink and parchment — was so high, and the monetary systems so inconsistent, that only in the case where transactions were incredibly large, could we imagine to record them.

Because of he first prosperity times in the mankind history in the area between the Tiger and the Euphrates rivers there was a need to record transaction. This need was early codified. For instance in the Hammurabi code, it was required that an agent who was selling a good in name of his principal gave him a price quotation under seal, the failure to perform this obligation being an invalidation argument for the transaction. So, transactions were engraved in clay tablets which were safely kept until final outcome and the recycled for a new transaction.

Egyptians use for a while the same support until the introduction of the papyrus which permits easily to extend records. A profession of specialized scribes was organized. It developed an elaborate internal verification system whose honesty and credibility were enforced by royal audits which conduct to important penalty in case of irregularities.

Ancient greeks introduced to major innovations : first, they established public accountants [1] to impose the authority and control of the State ; Secondly, about 600 B.C., they introduced coined money which not only facilitated transactions but lighten bookkeeping operations by the introduction of a local common unit. Under those innovations, the banking system, which has existed since the old summerian times, reached a level never reached, allowing change and loan operations and even cash transfers on a scale never reach until this time.

The romans developed the first system which recurrently maintain, at the level of the households their daily receipts and the expenses in *an adversaria* or account book. Then, they aggregated monthly those statement in a cashbook known as *a codex accepti et expensi*. All those operations, developed because the assets and liabilities of the citizens where the basis of the taxation systems and eventually were used to determine civil rights.

Then roman accountants developed an elaborate system of checks and balance for governmental receipts and spending which was necessary to fixed and verify all the operations links to a conquerors nation. Later on, when the empire was well established an accounting system was mandatory to keep record of all level of fiscal operations. To coordinate all the public financial operation, they conceive the first annual budgets.

But, at the time of the fall of the roman empire in 582, the necessary structures needed to teach how to write an account has vanished. One must know that romans use nearly the same educational system which was

[1] 10 chosen by lot.

nearly universally used in Greece. Until 12, young upper class boys stayed in their family where they received an education where the emphasis was set on letters, music and a great proportion of elementary arithmetic — essentially taught to know how counting either with the help of their fingers or with the abacus[2]. Then, they were normally send in a school where they learned literature, grammar, some elements of logic, rhetoric and dialectics.

Only those who needed a deeper mathematical teaching, as the one who planed to become *agrimensor*, that is surveyors, used to learn geometry. If they planed to become architect, they obeyed the Vitruve's advices to learn geometry, optics, arithmetic, astronomy and others fields as law, medicine, music, philosophy and history. Galen gave some nearby advices for student in medicine... The reason why mathematics where not high ranked in the roman education was essentially that it was useful only for liberal professions when the royal road in education conduced to public functions. It seems that, according to the accounting standards, quickly described above, this mathematical education was in all ways sufficient.

During the *dark ages*, because to learn you need peace, there was a fall in eduction in Europe to the unique exception of the british islands which, because of their insolation, were protected against the quasi-universal chaos. Of course, the higher centers of learning where rare but we do have testimonies of the transmission of the roman educational system. Toward the 9th century, in nearly every monasteries schools were organized to permit the acquisition of the christian culture to those destined to the priesthood or to the monastic life. But, because of the intended object of this eduction, the studies were limited to reading, writing and the study of the Bible. Only in very rare places like the cathedral of York, mathematics were taught[3].

Horrified by the low standard of education in continental Europe, Charlemagne, asked Alcuin to leave York to organize a court school with the posted object to permit at least to the clerk to interpret the Holy Bible correctly.

[2]It is not clear if the abacus is an roman invention or if it is a chinese one which come in occident as the silk cloth through the trade of the red sea and the coasting navigation along india.

[3]In 732, Egbert was bishop and head teacher of the York school. He organized the studies in such a way to teach rhetoric, law — essentially canonic —, physics, arithmetic, geometry. As the date of Easter was changing according to the moon, some arithmetics where also taught.

At his death, with the return of war, the educational level drop against until Gerbert was elected Pope under the name of Sylvestre II in 999. Gerbert was himself a mathematician who discovered some interesting document, so he favored the study of Boece, one of the rare roman who studied mathematics in the 5th century. But, as Boece was himself interested by the work of the surveyor, the mathematics studied had a more pronounced flavor of geometry than the needed arithmetics necessary to keep the accounts.

But, at the end of the dark ages, the english church fighting against the pagan influence, decided that mathematics had a too much paganist flavor and mathematics education against vanished. Because of all this sad history, in those times, only a very small bunch of peoples where able to go further that counting on the hand fingers.

Then at that time, there has been a reversal in education. In the british islands, it stagnated. Of course, between the 12th and the 14th century, many great universities were created in all Europe and the teaching of the mathematics of the ancient rise — even new mathematics were developed and diffused as the one of Fibonnaci[4] whose best promotor was Johannes de Sacrobisco who learned and teach in Paris and wrote his *Tractatus de Sphaera* whose stayed one of the most learned astronomic book until the end of the Renaissance. In those days, the first translation in Latin of the *Elements* of Euclide was done, and at least in the 14th century, the mathematical curriculum included algorithmic, ptolemaic astronomy, perspective, proportion, measurement of surfaces and... fingers accounting which was a pre-requisite to the entry in the Universities.

As more and more clergymen became mathematical educated, helped by the fact that Dominicans, who were more involved in education than older orders, attracted more and more poor young people eager to become monks, the mathematical education rise at all levels even if the used methods of teaching conducts more to rote learning than true understanding[5].

During all those times, the far most advanced mathematical teaching was done by the trade guilds. The apprenticeship lasted seven years : a master of a trade then taught to an apprentice all he thought he should know.

[4] His *Liber abbaci, Practica geometria* has been written between 1202 and 1228.

[5] Pupils were obliged to learn the question they could ask to the teacher and to learn also the answers.

Of course, those studies were purely practical and no apprentice could have learned more that what artisans and merchants could teach classes had he wanted to. Architect and builders but also merchants and traders and in the big cities the early forms of money landers use to learn geometry and arithmetics.

So from the trades, because they imposed the study of mathematics to their practitioners, came the conditions for a rapid advance in accounting technology which will be achieved during the Renaissance.

We must underline also that if in 976, the *Codex Albelendsis seu Vigilanus* was the first document to use arabic numeral, it take nearly 300 years before Leonardo Fibonacci, in his *Liber Abbaci*, advocated their universal use in replacement of the roman numerals[6]. But, and this could explain why trades used them so early, the spreading of their use come from the play of cards whose origin, is uncertain but likely situated in orient or in the middle-east. In the primitive literature on the subject, it is postulated that they were introduced in Europe by the arabs.

From playing card with arabic numeral to bookeeping with the same medium the step was natural for merchants who rapidly could not use two counting device, and the rise of the commercial relation of cities like Venezia or Genova was a good incentive to develop a new accounting system.

This new accounting system was finally explained and advocated by Luca Pacioli whose fame is shown in its enigmatic portrait[7]. As a Renaissance man, Pacioli accepted the interrelatedness of all the subject under study in his time : religion, business, military science, mathematics, medicine, art, music, law, language. He finally acquired a high level of knowledge in all those fields but he cherished the most those which exhibited harmony and balance as mathematics and accounting — see Alexander (2002).

If Pacioli is often credited to be the father of double entry bookkeeping, he never claimed for himself its invention. The most part of the authors who write on this subject argued that double entry bookkeeping was a common practice since the 13th century italian cities but there are rare exceptions as Kats (1930) who convincingly argued that it was a common

[6]The importance of the Fibonacci advocacy wa early recognized by the italian cities. As a proof, he receive a permanent income from Pisa, certainly to teach the arithmetics.

[7]For an analysis of the portrait painted by Jacopo de Barbari in 1495 which is exposed in many internet sites, it is worth to read McKinnon (1993).

practice in Rome long before this time or Lauwers & Willekens (1994) who refers to Colt (1844) whose thesis is that italians pick up their knowledge of double entry bookkeeping at Alexandria, Constantinope or some other eastern cities[8].

But many accounting historians, as Roover (1955), do not accept double registration of a transaction, on time in credit and one time in debit, as a sufficient condition to qualify the accounting system under study as a double entry system. For instance, de Roover insists on the fact that all transactions be twice recorded. *"This principle involves the existence of an integrated system of accounts, both real and nominal, so that the books will balance in the end, record changes in the owner's equity and permit and permit the determination of profit and loss"*.

With this in mind, according to Roover (1955), the oldest discovered record of a compete double-entry system is the one of the *Messari* — the *treasurers* — accounts of the city of Genoa in 1340 because not only it contains debits and credits journalised in a bilateral form, but each transaction is recorded twice in the ledger.

Pacioli himself credited the first description of the system to Benedetto Cotrugli, a Raguze merchant, who has written a book *Delai Mercatura et del Mercante Perfetto — Of Trading and the Perfect Trader* — which has been published a century later, but of which he was familiar.

The *Summa de Arithmetica, Geometria, Proportioni et Proportionality — Everything about Arithmetic, Geometry and Proportion* — has been written as a digest and guide to existing mathematical knowledge and bookkeeping was only one of the five topics covered. The presence of bookkeeping in this master piece and the fact that 37 short chapters entitled *De Computis and Scripturis*, were devoted to its study, acknowledged the fact that for Pacioli it was a major and perfectly legitimate mathematical subject for his time, a true algebraic application.

But, not only for Pacioli was it a major subject. The proof come from the fact that it has not circulated as manuscript copies but as printed copies

[8]Recent investigations tend to confirm this hypothesis. For instance, Albraiki (1994) proves that at the beginning of the Mamluk period between 1250 and 1517, double entry book keepin was already in use in Egypt and Syria. More than that, the old *Cairo geniza collection — geniza* meaning burial —, a collection of more than 200000 fragments found in the Ben Ezra synagogue during its restoration in 1890, contains a fragment dated from 1080 in the form of a journal and a four page account dated from 1134 listing both credit and debits — see Scorgie (1994).

— as soon as it has been finished, it has been directly printed in 1494 from the Paganino de Paganini printing house. Yet, even if the fact to print drive the reproduction price to a low level in comparison to the manual copy, in that times, printing could be achieved only to a high cost, and as the invention was only in its thirteen decade, it must have been a recognized urgency, to print such a book when so much manuscript where waiting to be printed. In all the cases, the book is an *incunabulum* that is to say a immeasurable value book printed when printing was still in its cradle.

More than that, in choosing to print his master piece in Venice where he came to accomplish this project which is demonstrated by the fact that since 1486, he visited main courts and lectured mathematics at various Italian universities such as Perugia, Florence, Rome and Napels where he taught also military science when he occupied no recorded function in Venice, Pacioli was certainly aware of the fact that he could discuss with some masters in accounting in the case where it happen to be necessary, and that it was the only place in the world where he was covered by some author rights since they have been invented by the Serenissima in 1474, even if it was only for the venetian states and for a 10 year duration. Even if this copyright give no protection against the copy by hands, there is no known hand copy of the *Summa*. It could seems strange because there was an abondance of scribes, whose work will last until the end of the 16th century because the strong resistance from some bibliophiles who prefered to possess an handwritten unique manuscript. According to Sangster, Stoner & McCarthy (2007), the lack of pirated copies of the *Summa* could be explained in part by the presence of diagrams and marginal notes which could make the copy relatively unattractive.

And, as Padova was in the venetian states, and because in Padova the University was independent of the Papacy, liberal fields were taught opening a large market for the book which nevertheless has been written mainly for the merchants[9].

The commercial success of the book was such that it was printed two times the same year and that his publisher Paganino de Paganini signed again Pacioli for two others books which were published in 1509[10]. In 1523,

[9]Sangster et al. (2007) argue that because the book has no worked exemples, he has been written for merchants who need not them.

[10]*De divina proportione* which was controversial in the sens where the third part is a translation in italian of the treatise on the five regular solid written by Pierro della Francesca and the translation of the *Elements* of Euclides.

the son of Paganini published a new edition of the book.

In what concern the number of printed copies of the *Summa* a first estimation of 300 printed copies by Antinori (1980) has been disallowed by Sangster et al. (2007) on the basis that it does not take into account the size of the print-runs of the late 15th century. On this basis, in their opinion it would be reasonable to infer that at least 500 copies were printed. But other factors, indicate that a greater number of copy could have been printed.

First of all some pages have been independently printed. It was the case in 1502 and in 1509 certainly, in the case of the first date, avoid the expiration of the 10-year copyright and in the case of the second one to take advantage of a 15 year copyright witch have been granted to Pacioli himself.

After some very convincing arguments, partially coming from the fact that the editor financed the publication of the 1523 second edition, Sangster et al. (2007) arrive to the conclusion that more than 1000 and perhaps up to 2000 copies of the *Summa* where sell.

2.2 The accountancy group

In accountancy an double-entry account can be defined as an ordered pair of number $(d, c) \in \mathbb{Z}^2$ where d is a debit and c a credit.

$$(d, c) = \frac{\textsc{Debit} \quad \textsc{Credit}}{d \quad \Big| \quad c}$$

In a newtonian accountancy, the gap between c and d is the balance or, in a more economic mood, the profit. In a newtonian world, we can add two accounts in such a way that if (d_1, c_1) is the first account and (d_2, c_2) is the second, we have : $(d_1, c_2) + (d_2, c_2) = (d_1 + d_2, c_1 + c_2)$. We can also add three accounts in such a way that if the first is (d_1, c_1), the second (d_2, c_2), and the third (d_3, c_3) we will have $((d_1, c_1) + (d_2, c_2)) + (d_3, c_3) = (d_1, c_1) + ((d_2, c_2) + (d_3, c_3))$. We can also remark that as $(0, 0) \in \mathbb{Z}^2$, $(d_1, c_1) + (0, 0) = (d_1, c_1)$. In other terms, in accountancy, $(0, 0)$ a neutral element.

Since, if $(d_1, c_1) \in \mathbb{Z}^2$, $(-d_1, -c_1) \in \mathbb{Z}^2$, in accountancy each accountancy has obviously an inverse because $(d_1, c_1) + (-d_1, -c_1) = (0, 0)$. And last but not least, the order in which one add the accounts has no consequences because $(d_1, c_1) + (d_2, c_2) = (d_2, c_2) + (d_1, c_1)$.

In the mathematical terminology, an euclidian accountancy is associative, it possess a neutral element and each account has an inverse and the addition of two accounts is commutative. In short, an accountancy is what mathematicians call an *additive abelian group*.

This as been noticed by Ellerman (1986) and incidentally by Lim (1966). Ellerman call it the *Pacioli group*[11]. As he remarks, a century earlier, the great Arthur Cayley — voir Cayley (1984[1896]) — has noticed by himself that if mathematicians do not seriously looked to accountancy as a mathematical object it is because of its apparent simplicity[12],[13]. Cayley was also the first to linked double-entry book-keeping to the euclidian ratios :

> *The Principles of Book-keeping by Double Entry constitue a theory which is mathematically by no means uninteresting: it is in fact Euclid's theory of ratios an absolutely perfect one, and it is only its extreme simplicity which prevents it from being a interesting as it would be otherwise* — Cayley (1984[1896])

But each mathematical presentation of group theory display conjointly an additive and a multiplicative group. So why is there no non-newtonian accountancy based on the multiplicative one. Even there is only one likely answer to this question which is to say that our brain is delivered only to compute additions and substractions, all other operations being acquired which give a great advantage to addition over multiplication, one must study the possibility of a multiplicative accountancy on the basis that the conception of the modern accountancy system as been a very time consuming process and that nothing could have prevent this system to have a distinct look from the one it finally takes.

If we call multiplicative accountancy a *accountancy it must operate on an ordered pair $(d, c) \in \mathbb{N}^2$. In this non-newtonian system the ratio between credit and debit — *i.e.* : d/c — becomes the profit. We can multiply two accounts in such a way that if (d_1, c_1) est the first account and (d_2, c_2) is the second, we will have $(d_1, c_1) \times (d_2, c_2) = (d_1 d_2, c_1 c_2)$. From here, it is straight to prove that an *accountancy is commutative, *i.e.* $(d_1, c_1) \times (d_2, c_2) = (d_2, c_2) \times (d_1, c_1)$. We can also multiply three accounts (d_1, c_1), (d_2, c_2) and (d_3, c_3) and find that this is an associative operation — *i.e.* : $((d_1, c_1) \times (d_2, c_2)) \times (d_3, c_3) =$

[11] Ellerman use the notation $[c||d]$ because it seems that it has been suggested by Pacioli himself.

[12] Cayley has been a lawyer for 14 years.

[13] Augustus DeMorgan is the only other great mathematician who was interested in accountancy — see DeMorgan (1869).

$(d_1, c_1) \times ((d_2, c_2) \times (d_3, c_3))$. The account $(1, 1) \in \mathbb{N}^2$ plays the role of the neutral element for the ⋆accountancy since $(d, c) \times (1, 1) = (d, c)$. Each account (d, c) has an inverse since $(d^{-1}, c^{-1}) = (1, 1)$.

As far as the newtonian accountancy is an abelian group, the non-newtonian ⋆accountancy is also an abelian group — Ellerman (2010) call it a *Pacioli multiplicative group* by contrast with the newtonian accountancy which is a *Pacioli additive group*.

To help to understand how one can substitute the Ellerman group — to give it a name — to the Pacioli group in double-entry accounting, we can transfer one example developed by Ellerman from the later to the former. We start from a company whose initial balance sheet equation, which ever be the unit of account, is :

Assets	=	Liabilities	+	Equities
15		10		5

According to the notation used by Pacioli himself, one can rewrite such a balance as :

Assets	=	Liabilities	+	Equities
[15//0]		[0//10]		[0//5]

with this notation which must be read [Debit//Credit], the convention that each value is recorded to the side where it could take a positive value. So in writing [15//0], we truly speak of a profit of 15-0 = 15, in writing [0//10], we truly speak of a debt of 0-10 = 10 and in writing [0//10] + [0//5], we write an addition of the same kind of the vectorial addition — *i.e.* : [0//10] + [0//5] = [0//15]. Of course, we can write this in the accountant usual manner, that is to say by presentation of tables, but this is more space consuming.

Suppose now that the firm realizes three distinct operations :

① the use of 1.2 unit of input inventories charged directly to the equity account;

② the selling of the product of its activity added directly to the equity account for an amount of 1.5;

	Assets	Liabilities	Equity
Initial Balances	[15//0]	[0//10]	[0//5]
+ ①	[0//1.2]		[1.2//0]
+ ②	[1.5//0]		[0//1.5]
+ ③	[0//0.8]	[0.8//0]	
Final Balances	[16.5//2]	[0.8//10]	[1.2//6.5]
Reduced Form	[16.5//2]	[0.8//10]	[1.2//6.5]

Figure 1: The double-entry accounting under the Pacioli group

③ the refunding of a loan for a cost of 0.8.

By the double-entry mechanism, this lead to the table 1. Now look at how to manage the same entry under the Ellermans group. Clearly we have :

$$\text{Final Balance} = \text{Initial Balance} + \text{Journal}$$

If now we want to present the double-entry bookkeeping according to the Ellerman group we will have

$$\begin{array}{ccccc} \text{Assets} & = & \text{Liabilities} & \times & \text{Equities} \\ 15 & & 10 & & 5 \end{array}$$

Symmetrizing the Pacioli group notation, we will have:

$$\begin{array}{ccccc} \text{Assets} & = & \text{Liabilities} & + & \text{Equities} \\ [50/1] & & [10//1] & & [5//1] \end{array}$$

where [50/1] is now taken for the income and is equal to $50 \div \div 1 = 50$, when [1/15] is taken as a debt and is equal to $1 \div 15 \approx 0.06667$. With this particular convention, the presentation of the same operations as before is given in the table 2.

We can easily verify that $78.125 \div (12.5 \times 6.25) = 1$. Whichever be the properties of the non-newtonian double entry bookkeeping based on the Ellerman group, it gives a coherence to the operation at the same level that standard or newtonian double entry-bookkeeping.

	Assets	Liabilities	Equity
Initial Balances	[50//1]	[1//10]	[1//5]
× ①	[1//1.2]		[1.2//1]
× ②	[1.5//1]		[1//1.5]
× ③	[1//0.8]	[0.8//1]	
Final Balances	[75//0.96]	[0.8//10]	[1.2//7.5]
Reduced Form	[78.125//2]	[1//12.5]	[1//6.25]

Figure 2: The double-entry accounting under the Ellerman group

What precedes, conduce to envision that one can use other balance equations in the economic analysis according to what one desire to emphasize. For instance, the consumer balance, which is by now written as:

$$W + \Pi = \sum_{i=1}^{n} p_i x_i$$

where W is the salary, Π the rent, x_i the consumption of the ith good and p_i its price, can rightfully be written :

$$W \Pi = \prod_{i=1}^{n} p_i x_i$$

In the same way, the profit equation for a firm — *i.e.* $\Pi = R - C$, where R is the revenue and C the production cost can also be written as :

$$\Pi = \frac{R}{C}$$

and, last but not least, the macroeconomic balance:

$$C + S + T = C + I + G$$

where C is the aggregate consumption, S the aggregate saving, T the taxes, I the aggregate investment and G the public expenditures could be written as follows:

$$CST = CIG$$

As we can see, the application of the Ellerman group to bookkeeping is not a mere curiosity since in writing of the balances which it generates, some ratios, which are often used in economics without so much justification, emerge naturally from them.

3 Conclusion

Finally we can conclude that ratios better compare two positive quantities than differences. This remark has been discussed by some Renaissance scholars and more recently by Grossman & Katz (1972) and since them an entire pleiad of mathematicians dedicated to the development of their seminal work. From their contributions, it follows that growth phenomenon are better described by the multiplicative point of view than by the additive one. As one can indifferently choose either the Pacioli or the Ellerman group from the bookkeeping point of view, and because the *accounting is better adapted to the description of multiplicative processes, we suggest that the *accountancy approach be used to express the balance sheets in the analysis of the economic growth, as was done in Filip & Piatecki (2014).

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