Expanding human capabilities: 
Lange’s Observations updated for the 21st century

Jorge Buzaglo

Abstract. Poland has produced two of the greatest economists of the past century, namely Michal Kalecki and Oskar Lange. Both worked with a wide and penetrating view of the economy and society, more typical of the great classical economists than of those of their own time. During the post-World War II “Golden Age of Growth,” while Keynes was the patron saint of economic theory and policy in the industrialized capitalist countries, Kalecki and Lange had a similar influence and role among the developing nations and — perhaps to a lesser extent — in socialist countries. Kalecki’s “The problem of financing of economic development” (Kalecki 1954), and Lange’s “Some observations on input-output analysis” (Lange 1957), in particular, deeply influenced the approach to economic and social development of a whole strand of structural economics, both in terms of economic analysis and practical policy. With the end of the Golden Age and the momentous neoliberal reaction then started, they have become almost forgotten. The economics of Kalecki, however, has inspired in the last few decades the renaissance of a genuine—albeit still marginalized—form of Keynesianism, the so called “post-Keynesianism.” (“Bastard Keynesianism,” as Joan Robinson called it, being the dominant, mainstream form of Keynesianism.) Oskar Lange, on the other hand, does not yet seem to have been re-discovered to inspire a similar renaissance of an advanced analysis and planning of policies for modern structural change and development. This paper proposes such a re-discovery. The main concern of Lange’s Observations is the appropriate intersectoral allocation of investment for efficient output growth. While output growth is still a main objective in most societies, the growth of human capabilities should be an increasingly relevant concern, both in its own right, and in view of the existing ecological constraints. Buzaglo (2014a, b) postulated the existence of a ℵ (aleph) matrix, describing the proportions of the different capabilities necessary for the achievement of every particular capability. The present paper is about introducing capabilities within Lange’s framework of analysis. A Lange-inspired analysis investigates the appropriate structures and properties of capability-enhancing growth paths.

Introduction
In recent times, many respected voices have been raised to point out that humanity confronts today a combination of several grievous, potentially catastrophic crises — economic, ecological, political, moral, social and cultural. It would seem as if the whole present system of convictions and behaviors is not viable and sustainable any more. A new paradigm of hopeful and inspiring ideas should be necessary for a new renaissance and a new period of positive development.

Economics is the central ideological constituent of the presently dominating paradigm. Mainstream economics is the model ideology of the avid, egotistic, myopic, insatiable material consumer. Mainstream economics is the proclamation and advertisement of an unsustainable and unviable paradigm. A new, more hopeful paradigm, I think, should shift attention and energies from the growth of material production and consumption to a different type of growth, namely human growth. Human growth should be understood as the process of “all-round development of individuals” (Marx/Engels), in which individuals are enriched by an all-round process of increasing aptitude in a growing domain of different exertions. Or, in the language of capabilities introduced by Amartya Sen, human development is the process of increasing the domain of human capabilities/liberties.

Oskar Lange’s Observations (Lange 1957) were one of the most powerful presentations of the detailed intersectoral characteristics of the process of growth of material production. They were useful to a whole generation of structuralist economists for their understanding of the process of

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economic growth and development. They were an important source of inspiration in policy modelling and planning.

Lange’s focus on the growth of material production was the relevant focus for a time of material penury and high aspirations of material welfare in most countries. The times are perhaps mature now for new aspirations and new paradigms, which would shift the focus towards the all-round development and flourishing of individuals and societies.

The analysis presented here departs from Lange’s approach in the Observations about the conditions for the expanded reproduction of the production system. In the second section, the system of production is linked with the system of capabilities through a matrix connecting investments in expanding capabilities with investments in the producing sectors. A short final section speculates about the possible characteristics of a system in which capabilities alone are relevant.

**Lange’s approach to investment and growth in an input-output framework**

For Lange, input-output analysis can be conceived as a development and generalization of the relations taking place in the process of reproduction of the national product, as studied by Marx in the reproduction schemes of Volume II of Capital.

Lange describes as follows the process of extended reproduction, when part of the social product is invested as means of production, and as a result, total output is growing:

The part of the net outputs of the various sectors invested in production is added to the means of production available in the next period. This makes possible in the next period an increase in the output of the various sectors of the national economy. The investment done in one period adds to the amount of means of production in operation in the next period. In consequence, a larger output is obtained in the next period. The outputs of successive periods (years, for instance) are linked up in a chain through the investments undertaken in each period. Thus, productive investment generates a process of growth of output. (Lange 1957, p. 210; page numbers from Nove’s (1964) edition of the work.)

This sentence may be given a formal expression:

\[ x_{t+1} = x_t + \tilde{x} d_t \]

(1)

In this equation, \( x \) is a dated \((n \times 1)\) vector of gross outputs, \( \tilde{x} \) is a \((n \times n)\) diagonal matrix of sectoral output-outlay ratios, indicating the effect of sectoral investment on output growth in the various sectors, and \( d_t \) is a \((n \times 1)\) vector of sectoral investments (i.e., investment by sector of destination). That is, \( x_{t+1} \), the outputs of the next period, are equal to the outputs in the current period \( x_t \), plus the increase in the output of the various sectors made possible by the investments \( d_t \) done in the current period. The increase in output is proportional to \( \tilde{x} \), Lange’s output-outlay ratios, more commonly known as incremental output-capital ratios. Lange (1957, p. 217) shows that in aggregate terms, “the rate of increase of gross national product is the product of the overall rate of investment and of the average output-outlay ratio.”

The simple causal structure of the model of equation (1) can be visualized in Figure 1.
Lange shows also that, departing from equation (1), “... we can calculate the effect of a given investment programme upon gross national income after a number of unit periods of time” (Lange 1957, p. 217). In effect, starting in the initial unit period in which outputs are known, and given the overall rates of investment, and the fractions of the total investment outlay allocated to the various sectors of the economy, we can from equation (1) calculate the trajectory of gross outputs over subsequent future periods. That is, given initial gross outputs $x_0$ and a time-sequence of sectoral investments $\{d_t\}$, equation (1) can be recursively solved forward in time, for known output-outlay parameters $\alpha$. In this way we determine the time-sequence $\{x_t\}$ of future gross outputs resulting from investment programme $\{d_t\}$. (Nowadays, this type of solution is called a simulation.)

When *Observations* was written there were very limited computing capabilities available, even in the most advanced economies, so the possibilities for these types of calculations were in general very limited. In a later work, based on his lectures at Warsaw University, Lange describes in certain detail the design of hydraulic or electrical models, or analogical computers, in order to study the solution of these potentially very complex numerical problems (Lange 1969, Appendix 1). He recognizes however that “...now that electronic computers are applied to numerical calculations, the possibilities of solving numerically complex economic problems are infinitely greater.” (Lange 1969, p. 132). Since then, the development of powerful programming and modeling systems such as GAMS or *Mathematica* has radically facilitated the implementation and solution of these types of problems.

The investments in the model of equation (1) must of course be consistent with the general structure of the economy, in particular with incomes generated in production, and with existent consumption and saving behaviors. In Lange’s model, largely describing the functioning of an ideal planned economy, the overall rates of consumption and investment are determined by the planning authorities, and consumption of the different categories of output is determined by behavioral (statistical) consumption parameters. In a capitalist market economy, or in a mixed economy, however, it may be relevant to incorporate a detailed description of the distribution of incomes among different size- or class-income groups (and the government), because different income groups have different savings and consumption behaviors. It may also be relevant for economic policy and investment strategy to have a detailed knowledge base for the design of appropriate redistribution policies for growth and equity.

A most convenient form for an operational income distribution function is a linear form:

$$y_t = V x_t$$  \hspace{1cm} (2)

in which incomes by income group $y_t$ (a column vector with $k$ files/groups), linearly depend on outputs $x_t$, according to proportionality coefficients given by the $(k \times n)$ matrix $V$. Matrix $V$ can be understood as a constant matrix in the case of distributional status quo, or it can vary according to planned or expected changes in income distribution — in this case we may have a time-sequence $\{V_t\}$ of income distribution matrices.

As said, a mixed economy version of Lange’s model would include an endogenous determination of consumption and savings according to behavioral parameters, at least for the private sector. The public sector may still influence the volume of overall savings available for investment through its influence on income distribution matrix $V$ — expanding/contracting public income and expenditures.
Consumption by type of output $c_t$ can be then described by a simplified linear expenditure system:

$$c_t = \Gamma y_t \quad (3)$$

where $\Gamma$ is a $(n \times k)$ matrix of consumption coefficients by income group and type of output.

The non-consumed part of total income is available for investment in the various sectors of the economy. That is, the volume of total savings, or investable funds in the economy equals incomes minus consumption. We write first savings by income class:

$$s_t = y_t - c_t^* \quad (4)$$

where vector $c_t^*$, like $y_t$, is a column $k$-vector, obtained from equation (3) of consumption by output category (an $n$-vector), by transposing, vertically summing, and diagonalizing matrix $\Gamma$:

$$c_t^* = \iota \Gamma y_t \quad (5)$$

in which $\iota$ is a summing vector $(1, 1, \ldots, 1)$ of appropriate dimension, the tilde $'$ denotes transposition and the hat $\tilde{\cdot}$ denotes diagonalization.

Now can saving by income class can be written as:

$$s_t = y_t - \iota \Gamma y_t = (I - \iota \Gamma) y_t \quad .$$

And, taking account of equation (2),

$$s_t = (I - \iota \Gamma) V x_t \quad . \quad (6)$$

Now, total savings in the economy is simply the sum $s_{\hat{t}}$, of the savings of all income classes that is, the vertical sum of $s_t$. The total available saving/investment fund $s_{\hat{t}}$ can be assumed to be centrally organized, as in Keynes (1936, p. 164), “on long views and on the basis of the general social advantage.” This can be interpreted as a central, direct allocation of sectoral investments, or as a mixed system, in which the state allocates a part of total investments (public investment in infrastructure, mixed enterprises, subsidized private investments, etc.), and the private sector allocates another part.$^2$

In the simpler case of direct, central allocation of investments, it is useful to think in terms of proportions of the common investment fund to be allocated to the different production sectors. We can define a distribution $n$-vector $z_t$, summing up to one, of proportions of the public investment fund allocated to the various sectors.

Sectoral investments in each period are then equal to the proportion corresponding to each sector times the total saving/investment fund:

$$d_t = z_t s_{\hat{t}} \quad . \quad (7)$$

$^2$ Buzaglo (1984, 1991, 2015) utilize a mixed-economy investment allocation system, in which public investment follows goals such as (re)industrialization or poverty reduction, while private investments are allocated according to an endogenous (accelerator) principle.
The dynamic loop is then closed with the increment in output produced by sectoral investment, according to the sector’s respective output-outlay parameter (equation (1)). A visual description of the process in Figure 2 may give a more intuitive picture of the process.

This model gives the pattern of sectoral gross outputs or supplies over time, but does not tell about sectoral demands. That is, “material balances” are not ensured in this description. In order to describe sectoral demand/supply balances we must be able to define intermediate demands, consumption demands, and investment demands. The following expression describes sectoral balances between supplies and demands:

\[ q_t = x_t - (Ax_t + c_t + Bd_t), \]  

(8)

in which sectoral excess demand \( q_t \) equals total supplies \( x_t \) minus total demands \( (Ax_t + c_t + Bd_t) \). \( A \) denotes the technical coefficients matrix, so that \( Ax_t \) is a column \( n \)-vector of intermediate demands. Consumption by type of output \( c_t \) was defined in equation (3) above. Given matrix \( B \) describing the sectoral composition (origin) of investments by destination \( d_t \), \( Bd_t \) is a column \( n \)-vector of investment demands by sector of origin.

In an open economy in which world prices prevail and output is tradable, sectoral excess demands \( q_t \) may be assumed to be traded; net exports when positive, and net imports when negative. An investment strategy may consist in the time-sequence \( \{z_t\} \) of investment allocation coefficients approaching as much as possible a desired time-profile \( \{q^*_t\} \) of exports and/or imports. This is the
case when for instance the consequences of some particular import-substitution, export-promotion, or industrial policy are investigated. Also, it can show the limits to overall growth and development imposed by agricultural stagnation, in turn caused by structural conditions in agriculture — in particular, property relations — as analyzed by the classic Kalecki (1954) article.

Lange introduces coefficients of labor requirements specific to every production sector, which implies that generally, different growth patterns will result in different levels of employment. Lange shows that an investment strategy attempting to maximize employment may be less successful (in expanding employment) than a growth-maximizing strategy after a number of time-periods. When it comes to the reduction of poverty, however, in some cases increasing employment as much as possible may be equivalent to minimizing poverty.³

**Extending Lange’s analysis to human capabilities**

We try now to approach the world of human capabilities from an economic point of view. It is a less well known world than the purely material world of production, and observations and opinions on the subject are (even more) conjectural than observations and opinions about the usual material world of economics. Since several decades, and particularly thanks to the work of Amartya Sen (see e.g. Sen 1988, 1999), the concept of development refers in economics more and more to the idea of expanding the domain of human capabilities, and less and less to the idea of increasing the flow of commodities. Human development can be understood as increasing the number and the scope of the different capabilities (faculties, aptitudes, abilities, capacities) that every member of society is able to exert. In principle, the space of human capabilities is a (potentially) infinite-dimensional space, that accounts for every conceivable dimension of human activity. In reality, however, we have relatively little knowledge of this potentially limitless space — although Nussbaum (2011) is an important contribution in this sense. In Buzaglo (2014), a tentative and conjectural effort is made to describe the possible structure and content of the capability space in more detail, including the possibility of constructing aggregate indexes over individuals and capabilities, thus suggesting a potential alternative to the GDP index. More specifically, it is suggested that it is possible to organize capabilities in a kind of hierarchical order, in which some basic capabilities are to be acquired or exerted before it becomes possible to exert other less basic capabilities, and so on. In addition, if the proportions in which every capability is used in the formation of every other capability are knowable, then something similar to the technical coefficients matrix of input-output analysis could be conceived, which we called the $\aleph$ (aleph) capability matrix. As in the case of dynamic input-output theory, the $\aleph$ matrix, together with a matrix of “capital capabilities” $\beth$ (beth) may be used to describe the potential or ideal equilibrium capability growth rates inherent in the system.

Following Lange (1957), we adopt here a somewhat more realistic simulation approach. Also, we assume a mixed or transitional economy, in which the recognition of capability structures and the search for fulfillment of expanding capability goals coexist with the material production system, including its standard processes of production, income generation and distribution, consumption, saving and investment.

Hence, we have two parallel systems, namely, the regular production system with the growth equation of (1) above,

³ This is shown in the case of Bolivia (Buzaglo 2015, Ch. 6).
System I: \[ x_{t+1} = x_t + \alpha^I d^I_t, \] (9)
in which the \( I \) superscripts refer to output-outlay ratios and sectoral investments within the standard production System I.

System I coexists with, and is parallel to, System II, which describes the growth of capabilities (we continue to use Hebrew letters for elements in the capability space):

System II: \[ v_{t+1} = v_t + \alpha^{II} d^{II}_t \] (10)

where \( v \) (ayin) is a dated \( m \)-vector of capability indexes, \( \alpha^{II} \) is a diagonal \( m \)-dimensional matrix of incremental capability-outlay ratios, indicating the increase in the capability index associated with a unit investment in that capability category. Investments \( d^{II}_t \) refer to investments in the \( m \) different capability-creating activities.

Now, there is a link between systems I and II; we assume that a \( (n \times m) \) matrix \( \psi \) (shin) exists, which describes the composition in outputs (System I) of the investments realized in capability-creating activities (System II). Matrix \( \psi \) describes how investments in the capability System II are composed of investment goods and services from production System I:

\[ d^I_t = \psi d^{II}_t \] (11)
or also, if we assume that an inverse of \( \psi \) does exist:

\[ d^{II}_t = \psi^{-1} d^I_t \] (12)

Our new recursive growth equation for capabilities, taking account of equation (12), becomes:

\[ v_{t+1} = v_t + \alpha^{II} \psi^{-1} d^I_t \] (13)

Which means that by regulating investments \( d^I_t \) in the production System I it is possible to influence the evolution of capability System II.

A possible aspiration for a given society, for instance, may be to achieve, within a given time horizon \( T \), a certain \( v^* \) structure and level of capabilities — e.g. certain proportions of basic, intermediate and high capabilities. The problem may be then specified as finding, with start in \( t=0 \), the time-sequence of investment distribution coefficients \( \{z_t\} \) that make the difference between real and desired structures as small as possible in period \( t=T \):

Find \( \{z_t\} \) such that \( \|v_T - v^*_T\| = \min. \) (14)

Another type of aspiration would be for instance to attain the maximum feasible level of an index of capabilities \( f(v) \), reflecting the “general social advantage” — that should today include some definition of ecological advantage:

Find \( \{z_t\} \) such that \( f(v^*_T) = \max., \) or also,

find \( \{z_t\} \) such that \( \sum_{t=0}^{T} v_t = \max. \)
In Figure 3, System I shows the growth of outputs \( x_t \), and System II describes the growth of capabilities \( y_t \). The growth of both systems is dependent on the level and structure of investments in production System I, \( d^I_t \). But \( d^I_t \) can now be regulated so as to achieve different desired capability expansion objectives—i.e. different functions of \( y_t \).

Conceiving a post-transitional mode of capability expansion

We try now to conceive how to consider capability growth in an environment in which output growth and the output producing System I has lost much of its relevance, for instance because of the achievement of a state of relative material non-scarcity, and/or because of changes in the socio-psychological capacity for the cognition of satiety. In such an environment, System I is a self-regulating, homeostatic provisioning system, and System II alone is relevant for analysis. We are now interested in the growth of capabilities, according to different possible, socially determined patterns. It is no longer necessary to introduce monetary categories, such as the investment outlays in the above analysis. We can think in terms of the amounts of time dedicated to the different capability-
creating activities. (These activities need not be thought of as creating painful “disutility,” but as different forms of largely gratifying, de-alienating forms of expression of immanent creative powers.) We can call this new, exclusive capability system, System III. In System III, the expansion of the different capabilities depends on the amounts of time destined to capability-increasing activities within each activity sphere (such as e.g., the innovative use of existing or new knowledge, or the induction of cross fertilization among activities), and on the respective capability index response to the time allocated to them. Formally,

$$u_{t+1} = u_t + \alpha^{III} d^{III}_t$$

in which capabilities in the following period $u_{t+1}$ depend on capabilities in the current period $u_t$, plus the additional capabilities created through time investments in capability-creating activities $d^{III}_t$, times the incremental capability-to-invested-time ratios $\alpha^{III}$.

**Conclusion**

Lange’s (1957) *Observations* were an enlightening contribution to the comprehension of the process of growth of material production. The ecological crisis, and the general crisis of the dominating paradigm of material production, suggest the prospect of a paradigm shift, and the convenience of reorienting economies and societies towards the development of human capabilities. It is suggested that the focus of policies and institutions should now turn towards the all-round development and flourishing of individuals and societies. Our paper tried to show that the insights of the *Observations* may serve to shed some light on how to conduct a transition towards such a new focus. These insights may even suggest possible modes of operation for post-transitional, capability-based economies.

**References**


